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ABSTRACT

The researcher conducted a six-month ethnographic study as an apprentice carpenter in Cape Town, South Africa, to document the valid mathematical ideas that are embedded in the everyday woodworking activities of a group of carpenters. A secondary objective was to examine and to give a firsthand account of the teaching and learning of mathematical ideas in the context of the researcher's apprenticeship. Finally, the study offers methodological techniques for identifying mathematics in thought and action and for differentiating the mathematics from routine applications of procedures. The results showed that many conventional mathematical concepts are embedded in the practices of the carpenters. They made extensive use of such concepts as congruence, symmetry, proportion, and straight and parallel lines in their everyday work. Furthermore, the carpenters' problem solving was enhanced by their strength in spatial visualization. Their explanations, in the form of convincing arguments, showed the sequential, logical reasoning that is related to the need in mathematics for proof and substantiation. The results also showed that the carpenters' mathematics has several unique characteristics: there was tacit mathematical knowledge in their actions, and reflection on actions led them to articulate their tacit knowledge; decontextualized questions posed were revised into concrete, contextualized problematics; and their ideas were framed by the context of the workshop and carpentry tools. Comparison, using the senses of touch and sight, was preferred to measuring and usually resulted in optimal solutions. To solve a problem such as "How many table legs can be cut from this plank?" spatial visualization practices were used to construct functional units, producing an optimal result that could not be obtained with formal procedures. The results are presented as a series of 20 narrative episodes, followed by an analysis. The epistemological, educational, and methodological implications of these results are discussed. (Contains over 100 references.) (Author)

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An Ethnographic Study of the Mathematical Ideas of a Group of Carpenters

by

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University of Northern Colorado
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DEDICATION

This work is dedicated to all those in South Africa who have struggled and who still struggle for a worthwhile education, for freedom from oppression, and for a just system of government.

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ABSTRACT

The researcher conducted a six-month ethnographic study as an apprentice carpenter in Cape Town, South Africa, to document the valid mathematical ideas that are embedded in the everyday woodworking activities of a group of carpenters. A secondary objective was to examine and to give a firsthand account of the teaching and learning of mathematical ideas in the context of the researcher's apprenticeship. Finally, the study offers methodological techniques for identifying mathematics in thought and action and for differentiating the mathematics from routine applications of procedures.

The results showed that many conventional mathematical concepts are embedded in the practices of the carpenters. They made extensive use of such concepts as congruence, symmetry, proportion, and straight and parallel lines in their everyday work. Furthermore, the carpenters' problem solving was enhanced by their strength in spatial visualization. Their explanations, in the form of convincing arguments, showed the sequential, logical reasoning that is related to the need in mathematics for proof and substantiation.

The results also showed that the carpenters' mathematics has several unique characteristics: there was tacit mathematical knowledge in their actions, and reflection on actions led them to articulate their tacit knowledge; decontextualized questions posed were revised into concrete, contextualized problematics; and their ideas were framed by the context of the workshop and carpentry tools. Comparison, using the senses of touch and sight, was preferred to measuring and usually resulted in optimal solutions. To solve a problem such as "How many table legs can be cut from this plank?" spatial visualization practices were used to construct functional units, producing an optimal result that could not be obtained with formal procedures.

The results are presented as a series of 20 narrative episodes, followed by an analysis. The epistemological, educational, and methodological implications of these results are discussed.

Chapter 1

INTRODUCTION

Overview

Mathematical knowledge has commonly been portrayed as consisting of universal truths that exist independently of people and that are discovered by mathematicians through a process of formal reasoning. Mathematical reasoning, unlike any other cognitive activity, is believed to be a decontextualized activity, tied to a formal system that relies upon a specifically defined set of symbols. These ideas have led to a view of mathematics as divorced from ordinary human activity and devoid of social, cultural, and political considerations.

The social, cultural, and political aspects of the development of mathematical ideas have consistently been overlooked in historical reconstructions of the growth of mathematics. Contributions from diverse non-Western cultures have been underrepresented in prejudiced historical accounts, and it has long been commonly accepted that most of the ideas referred to by the label "mathematics" emanate from Western civilizations (Joseph, 1987). The extent to which people have bought into the dominant Western view of what counts as mathematics is brought into sharp focus by reflecting upon the remarkable uniformity of school mathematics curricula throughout the world. In the analysis of the mathematics curricula of 22 education systems from countries around the world (e.g., USA, Japan, Hungary, Thailand, Nigeria, Scotland, Swaziland), the International Association for Educational Achievement has reported that there is a substantial "common core" in most areas of mathematical content across curricula (Travers & Westbury, 1989). Although this may be deemed desirable by those who wish to set international standards for achievement in school mathematics courses, such uniformity unfortunately emphasizes the formal and decontextualized and ignores the relevance and enrichment that cultural and community practices could bring to individual curricula.

From the point of view of anyone holding a formalist conception of mathematics, the activities of learning and teaching mathematics must be concerned with coming to know, or helping someone else come to know, the rules and conventions of the universally accepted formal system and the strategies that can be brought to bear on problems, so as to discover answers that are supposedly universally correct. In following this model of teaching and learning mathematics, the authority and expertise lie with the teacher, and the emphasis is on the students' absorption of already existing ideas. Little room is allowed for students' own informal conceptions of mathematical ideas. Mathematical knowledge generated in contexts outside of classrooms is usually not acknowledged as relevant to the academic setting, since such knowledge may not conform to the tenets of what counts as "genuine" mathematics according to the interpretation of teachers. Steffe (1986) comments that "the formalist view of mathematics *encourages* mathematics teachers to become mathematically inactive," so that their search for mathematical meaning "can begin and end with the formal, symbolic presentations of rules and procedures" (p. 3). As a result, Steffe suggests that such teachers may be shocked to learn that the rules for the multiplication of signed numbers that they have portrayed as unassailable truths are actually conventions.

As a consequence of unhappy childhood experiences in the mathematics classroom, many people experience a sense of alienation, coupled with fear and despair, at the mere mention of the word "mathematics." The National Research Council's (NRC) recent report to the nation on the future of mathematics education presents a dismal picture of the current situation:

> Mathematics is not seen as something that people actually use, but as a best forgotten (and often painful) requirement of school. For most members of the public, their lasting memories of school mathematics are unpleasant—since so often the last mathematics course they took convinced them to take no more. (NRC, 1989, p. 10).
>
> Three of every four Americans stop studying mathematics before completing career or job prerequisites. Most students leave school without sufficient preparation in mathematics to cope either with on-the-job demands for problem-solving or with college requirements for mathematical literacy. (NRC, 1989, p. 1).

My deep concern about the widespread failure of mathematics education gave impetus to the present study. Over 25% of all high school students in the U.S. drop out before graduating; among black, Hispanic, and native American students, the dropout rate often exceeds 50%. As observed in the NRC study, "Mathematics is the worst curricular villain in driving students to failure in school. When mathematics acts as a filter, it not only filters students out of careers, but frequently out of school itself" (NCR, 1989, p. 7). In South Africa, mathematics presents a similar stumbling block to many students, in particular to black students.

I believe that the failure of mathematics education is due in part to the sterile, narrow description of what counts as authentic mathematics. Many people feel themselves to be excluded from participation in genuine mathematical activity. When an individual is faced with a mathematical problem to solve, it is not unusual to hear her[1] apologize for not using "real" mathematics as she proceeds to use an efficient and innovative (but not school-taught) method to reach a satisfactory solution (Lave, 1985). Mathematics education is further hindered by a belief that only a small proportion of humankind is capable of fully understanding mathematics. An elite coterie of professional mathematicians is seen to own the prized body of knowledge and to control the creation of new mathematical knowledge. Throughout history, the individuals admitted to the inner circle of mathematicians have been predominantly white males. Today, white men earn 74% of the doctoral degrees in the mathematical sciences awarded to U.S. citizens, while black men earn 5% of these degrees. Despite the fact that women earn 46% of all mathematics baccalaureates awarded, they earn only 20% of the doctoral degrees awarded (NRC, 1989, p. 22). Tersely stated, what counts as mathematical knowledge is narrowly defined, and the access to this knowledge is through an even narrower doorway.

---

[1] Although feminine pronouns will be used throughout for the sake of uniformity, they are intended to include the masculine case as well.

The Cultural, Social, and Political Components of Mathematics

I propose to argue for a broader conception of what counts as mathematics, drawing on the literature to support my claims that mathematics is *culturally based*, in that all cultures generate mathematical knowledge to suit the goals and purposes of communities who wish to solve problems and to impose order in their lives (Bishop, 1987, 1988a, 1988b; D'Ambrosio, 1985, 1987; Fasheh, 1982, 1988; Gerdes, 1985, 1986, 1988a, 1988b; Stigler & Baranes, 1988); that mathematics is *socially constructed* in the context of a community, where meaning is negotiated and conventions are agreed upon (Bishop, 1985, 1988a; Cobb, 1986a; D'Ambrosio, 1985, 1987; Gerdes, 1985, 1986; Lakatos 1976; Steffe, 1986; Tymoczko,1980, 1986); and that mathematics has a *political component* (Fasheh, 1982, 1988; Gerdes, 1985, 1988c; Harris, 1987, 1988a, 1988b; Kallaway, 1984; Mellin-Olsen, 1987). Access to mathematics worldwide is anything but democratic. Race and gender differences in mathematics participation and achievement are not consequences of free choice or of some innate lack of capability. Rather, these differences are the consequences of an imbalance of power and the destructive effects of oppression, authoritarianism, and social conditioning.

Mathematics in Everyday Settings: "Everyday Cognition"

Researchers who have recently investigated people's use of mathematics outside the confines of the classroom have begun a path that could lead to a less exclusive view of mathematics. The recent research into cognition in everyday activities has concentrated on people's use of mathematical knowledge, and it urges recognition of the importance of activity and enculturation to learning (Brown, Collins, & Duguid, 1989). The practice of mathematics has been explored in the contexts of a variety of everyday settings. Two main groups of researchers have explored the use of mathematics in settings outside of schools. They are those interested in "everyday cognition" or "cognition in practice," where Lave (1985, 1988) is a key figure, and those interested in "ethnomathematics," where D'Ambrosio (1985, 1987) and Gerdes (1985, 1986, 1988a, 1988b, 1988c) are key figures.

I shall refer briefly to the work of those researchers interested in "everyday cognition," broadly grouping together the works of Lave (1977, 1982, 1985, 1988); Reed and Lave (1979); Petitto (1979); Lave, Murtaugh, and de la Rocha (1984); Schliemann (1984); Scribner (1984, 1985); Murtaugh (1985); de la Rocha (1985); Carraher, Carraher, and Schliemann (1985); Carraher and Carraher (1987); Saxe (1988); and Schliemann and Acioly (1989). (For a detailed review of this literature see Chapter 2.) In general, the two main aims of this group of researchers were, first, to explore the transfer of school knowledge to out-of-school situations, and second, to critique conventional cognitive theory, by using social and cultural theory, and the theory of practice. In order to achieve these goals, the arithmetic practices of, among others, tailors, cloth merchants, child market vendors, dieting cooks, grocery shoppers, bookies, carpenters, and dairy workers were studied.

This genre of research provides valuable empirical evidence that people frequently construct and apply innovative and creative methods of their own to reach satisfactory solutions to real-life problems. Schools, therefore, obviously do not have the monopoly on teaching mathematics. The methods used to solve problems were chosen to suit the goals and purposes of the problem solvers; for instance, efficiency and labor-saving concerns shaped the strategies employed by the grocery shoppers, dieting cooks, and dairy workers. The work conducted in practical settings also provides evidence that mathematical knowledge is developed within a context and is framed by that context. For example, useful units were invented that made use of the social and physical environments to facilitate calculations: tailors in Liberia computed in "trousers' worth" of cloth, and dairy workers used case prices as units (Lave, 1985, p. 173).

A further valuable contribution from this work is the suggestion that an apprenticeship model is a viable model for teaching and learning, and not only for the teaching of practical trade skills as is the case traditionally (Lave, 1982). In an apprenticeship model, instruction is conceived of as a joint problem-solving event in which

a more expert individual provides support or "scaffolding" to extend the skills of a novice to higher levels of competence (Rogoff & Gardner, 1984). Learning is embedded in activities within social and physical contexts.

There are several ways in which the present study extends the work in "everyday cognition." In all the studies discussed in the preceding two paragraphs, the focus has been on arithmetic practices that are generated by people in their work or home environments. No attempts have been made to study geometry (except perhaps for the work of Scribner, who describes how dairy workers avoided using arithmetic by relying on spatial visualization to solve problems). In fact, Lave et al. (1984, p. 68) state that one of the several reasons for focusing on arithmetic was that "arithmetic activity has formal properties which make it identifiable in the flow of experience in many different situations," and Lave (1988, p. 5) states that "it (arithmetic) has a highly structured and incorrigible lexicon, easily recognizable in the course of ongoing activity" (Lave, 1988, p. 5).

I decided to make *geometry* the focus of my study for exactly the opposite reasons. I believe that the richness of geometry in our physical environments provides fertile opportunities for people to develop complex geometric knowledge that does not look anything like the formal geometry we learn in schools. An understanding of geometric concepts can be expressed in a variety of different ways, through an appreciation of the aesthetic in design and through the creation of a sense of balance in for example, art, architecture, or furniture design. D'Arcy Thompson (1961, p. 269) reminds us of Galileo's aphorism, "The Book of Nature is written in the characters of Geometry." Whereas arithmetic consists of procedures for carrying out calculations and has a limited vocabulary for describing those procedures, the conceptualization and expression of geometry is not similarly constrained.

Setting for the Present Research

In order to explore my intuitions and tentative formulations about how geometry and other mathematical knowledge may be constructed and used in everyday activities, it was

necessary to decide upon a suitable site for investigation and an appropriate methodology to guide the investigation. I chose to conduct fieldwork in a carpentry workshop in Cape Town, South Africa, with a group of carpenters who represented a diverse spectrum of South African cultural backgrounds and language groups. One particular furniture workshop was chosen as the site for fieldwork for several different reasons. The workmanship displayed in the furniture produced was of an extremely high quality. The workshop was not highly mechanized—much of the construction of the furniture was done with old-fashioned handtools according to the old traditions, and hand-made articles were highly prized by the group. The excellent quality of the designs, the skillful construction, and the fact that the carpenters created beautiful furniture from rough planks with little assistance from mechanized systems convinced me that they had to be using mathematical, and in particular, geometrical ideas to guide their activities. Only one of the artisans had received formal training at a technical school. The other artisans had learned the craft of woodworking in traditional informal apprenticeships, handed down from father to son, or simply "on the job" while working with more experienced carpenters. All of the carpenters' formal schooling had been interrupted between the ages of about 12 to 16 years, so the group had had little involvement with school mathematics.

Two questions were especially intriguing to me. What did the mathematics used to guide carpentry activities look like, and where did these ideas come from? In order to address questions of this nature, I had to evaluate the methodological options critically before choosing an appropriate methodology to guide my investigation.

Methodological Framework: An Ethnographic Approach

My examination of the "everyday cognition" literature gave rise to three other issues that served as further points of departure for the present study. The first issue is that the methodology used in the studies mentioned above could not challenge what should or should not be counted as mathematics. The general methodological approach was to conduct naturalistic observation of the subjects at work and to note their performance on the

arithmetic tasks encountered during the working day. Pencil-and-paper tests or verbal tests of arithmetic tasks of a similar, but "academic," variety were constructed and administered, and comparisons of performance were made between school arithmetic and work arithmetic. School arithmetic was thus used as a kind of benchmark. Second, no explicit attempts were made to examine the epistemological roots of the invented arithmetic (that looked different from school-taught arithmetic) with a view to presenting it as a viable alternative system. Lave (1985) commented, "There may be a qualitatively different organization of arithmetic in different settings; the procedures observed did not look, feel, or sound like school arithmetic performances" (p. 173). The analysis performed was not critical enough to provide evidence of alternative, valid concepts of arithmetic.[2] Third, most of the studies did not investigate the ways in which this arithmetic knowledge is taught and learned outside of schools.

These three points highlighted the need for the careful selection of both a methodological framework and an epistemological framework to guide my research. Since I want to argue for a broader conception of what counts as valid mathematics, it was important that I gather evidence that could be seen as a challenge to the conventional, narrow definition of mathematics. An ethnographic approach, as described by Eisenhart (1988), was selected as being the most appropriate methodological choice that would allow me to explore the kinds of issues I had in mind. Many of the tenets of ethnography derive from the interpretivist philosophical stance, which has at its heart the idea that "all human activity is fundamentally a social and meaning-making experience" (Eisenhart, 1988, p. 102) and that objects do not have any intrinsic meaning; rather, the meaning of objects

[2]For example, I argue that the carpenters display a valid understanding of the concept of volume despite the fact that they do not know how to calculate volume numerically (Episodes 15 to 19, Chapter 5). Their understanding of the concept is of a visual nature and is not linked to numbers. They are able to use their concept of volume to take actions and to solve complex problems. Their concept of volume is very different from that which is conventionally taught in schools, and if judged according to the academic yardstick, their understanding of volume would remain invisible.

"lies in the actions that human beings take toward them" (Denzin, 1978, p. 7). Interpretivism and ethnography provided me with the framework for understanding and interpreting the carpenters' actions and practices in terms of mathematics, without constantly and explicitly invoking conventional mathematics as a standard of comparison. Eisenhart (1988) sums up what she means by interpretivism:

> From this perspective, meanings and actions, context and situation are inextricably linked and make no sense in isolation from one another. The "facts" of human activity are social constructions; they exist only by social agreement or consensus among participants in a context and situation. What counts as marriage, gender roles, teaching, calculation, the right answer, or whatever depends on the ways (and whether) these things are defined and used in human groups. . .it makes no sense for the interpretivist to do things like catalog beliefs about mathematics without also considering the contexts in which these ideas are important. (p. 103)

Epistemological Framework: A Constructivist Approach

The constructivist theory of knowledge creation provided me with an epistemological framework around which I could start to make sense of coordinating the existence of different representations of mathematics (Cobb, 1985, 1986b; Confrey, 1985, 1987, 1991; Kaput, 1985; Steffe, 1986, 1988; von Glasersfeld,1982, 1984). Constructivism provides a model for the construction of mathematical ideas through repeated cycles of actions taken to resolve a problematic and reflections on those actions (Confrey, 1985, 1987). Confrey (1991) uses the term "problematic" to emphasize that in constructivist theory the problem does not have independent status; it is rather seen as a "roadblock" to where an individual wants to be. Thus "problematic" is used to refer to the individual's "roadblock." Constructivism encourages and acknowledges the legitimacy of students' own inventions and explanations, fosters the acceptance of multiple representations of ideas, and emphasizes the importance of action and reflection (for a more detailed discussion of constructivism, see Chapter 2).

Unfortunately, the English language does not have a word for the activity of "doing/making" mathematics. The only way in which we can refer to this multifaceted human phenomenon is via the noun "mathematics," which seems to refer to a static body of

knowledge that already exists out in the world. We need a word to express the concept of an active process of creating and using mathematical ideas or tools. I shall use the verb "mathematizing,"[3] which embodies action and refers to the experience of creating and using mathematical ideas.

Mathematics in Everyday Settings: Ethnomathematics

The second group of researchers who have explored the creation and use of mathematics in settings outside of schools are those interested in ethnomathematics. I have grouped together the work of Fasheh (1982, 1988), Bishop (1985, 1987, 1988a, 1988b), D'Ambrosio (1985, 1987), Gerdes (1985, 1986, 1988a, 1988b, 1988c), Ascher and Ascher (1981, 1986), Harris (1987), and Borba (1987a, 1987b). In general, the two main aims of this group of researchers have been to explore the mathematics that is created in different cultures and communities and to describe this mathematics. In order to achieve these goals, the mathematical practices of national groups, people living in shantytowns, fishermen, basket weavers, rug weavers, knitters, and seamstresses have been studied. This genre of research provides convincing theoretical arguments that different forms of mathematics result from different ways of thinking that exist across cultural groups. Practical, empirical evidence of this mathematics has also been provided—for instance, the quipus of the Incas studied by Ascher and Ascher (1981, 1986); the mathematics used by fishermen and peasants in Mozambique, described by Gerdes (1985, 1986, 1988b); and the mathematics of children in a shantytown in Brazil (Borba, 1987a, 1987b). This genre of research has often carried hints and warnings that there is also mathematics that we are not able to perceive. For instance, Ascher and Ascher (1986, p. 139) warn that "as Westerners, we are confined

[3]The term "mathematizing" has cropped up infrequently over the years. The earliest reference I could find was attributed to Hermann Weyl. Kline (1980, p. 6) and Davis and Hersch (1981, p. 60) quote Hermann Weyl, considered to be one of the greatest mathematicians of this century, as saying in 1944, "'Mathematizing' may well be a creative activity of man, like language or music, of primary originality, whose historical decisions defy complete objective rationalization."

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in what we can see and what we can express to ideas in some way analogous to our own." Fasheh (1988, p. 2), whose illiterate mother is a seamstress, observed:

> My mother's math was so deeply embedded in the culture that it was invisible through eyes trained by formal education. . . .The math she was using was beyond my comprehension. . . .It was a shock to me to realize the complexity and richness of my mother's relationship to mathematics. Mathematics was integrated into her world as it never was into mine.

The paradox embedded in ethnomathematics. My critique of this literature lies in the fact that it seems to embed an intriguing paradox to which little attention has been paid. The paradox unfolds as follows:

1. Ethnomathematics is concerned with the study of the different kinds of mathematics that emerge from different cultural groups.

2. It is impossible to recognize and describe anything without using one's own frameworks.

3. The paradox: How can anyone who is schooled in conventional Western mathematics "see" any form of mathematics other than that which resembles the conventional mathematics with which she is familiar?

A Strategy for Conducting Practical Research in Ethnomathematics

The paradox described above gave rise to a dilemma, and it became clear to me that in order to proceed with the exploration of the mathematics of an unfamiliar culture, I would have to navigate a passage between two dangerous areas (see Figure 1). The foundering point on the left represents the overwhelming notion that "everything is mathematics" (like being swept away by a tidal wave!), while the foundering point on the right represents the constricting notion that "formal academic mathematics is the only valid representation of peoples' mathematical ideas" (like being stranded on a barren desert island!). Part of the way in which to ensure a safe passage seemed to me to openly

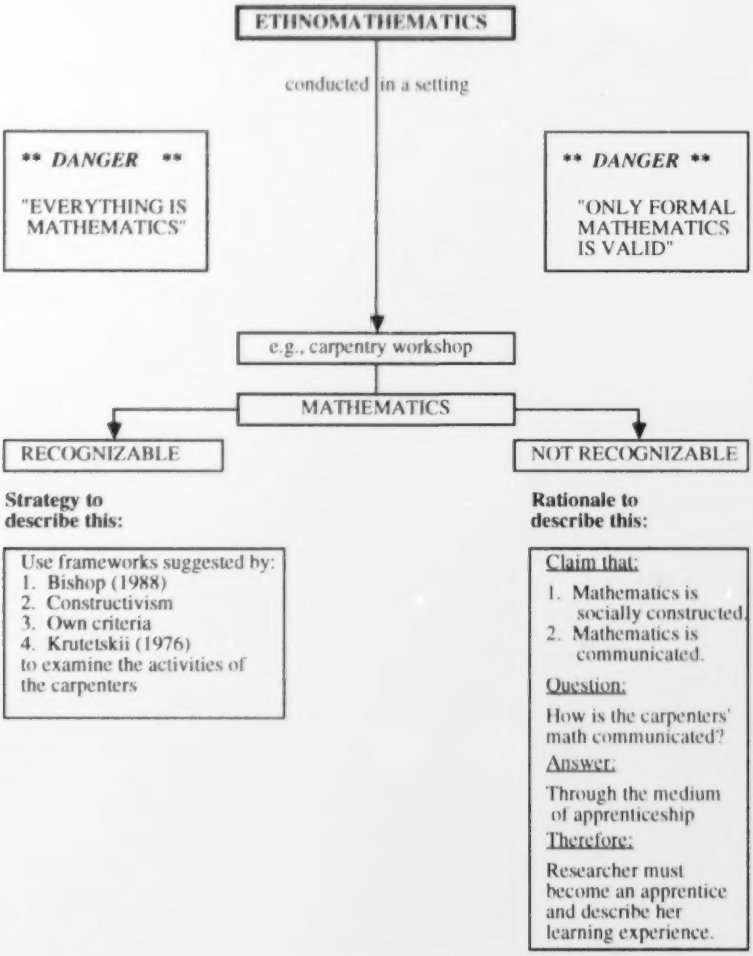

Figure 1. Scheme of strategy guiding present study.

acknowledge that when I examined the mathematizing engaged in by the carpenters there would be examples of mathematical ideas and practices that I would recognize and that I would be able to describe in terms of the vocabulary of conventional Western mathematics. However, it was likely that there would also be mathematics that I could not recognize and for which I would have no familiar descriptive words. A parallel situation is described by Becker (1989), who discusses the experience of learning a new language:

> Coming to know another language, particularly a distant one, confronts the knower with silences, things in one language which have no counterpart in another. These silences are often obscure because one fills them in as one actively hears (exuberances) or doesn't notice them at all because the distinctions don't exist in one's own language (deficiencies). (p. 14)

It was my aim to try to notice the silences with which I was confronted when coming to know the mathematizing of the carpenters and to find a way to give a voice to some of those silences, a voice in which the carpenters' *own* voices could be heard and valued.

I propose to set out below the rationale that I adopted to select, describe, and analyze each of these two categories of mathematizing, as a suggestion for how practical research in ethnomathematics may be approached. The conceptualization of describing two categories of mathematizing had several fortunate outcomes. The choice of ethnography as the most appropriate methodology was reviewed and refined, and from this choice emerged a strategy to explore the forms of mathematizing with which I would not be familiar. Spradley's comment (1980), "Rather than *studying people*, ethnography means *learning from* people" (p. 3), took on greater significance. I realized that the only way in which I could hope to find a way to understand the mathematics that would not be recognizable to my eyes (shaped by many years of studying and teaching formal academic mathematics) would be through my learning experiences as a carpentry apprentice.

My assumption is that the carpenters' mathematics is socially constructed and that it is communicated through the medium of apprenticeship. In order to experience the communication, it was essential that I should be a learner carpenter in the workshop. One of my conjectures is that tacit mathematical knowledge drives the physical activities of

designing and constructing furniture in the workshop, so it was vital that I come to know those physical activities myself. The knowledge that I would develop through the medium of action would give me an experience on which to reflect, both during the activity and afterward. From my reflection would emerge a way to describe my experience. To this end, I worked as an apprentice at the workshop on a daily basis for a period of five months, receiving instruction from senior artisans on an informal basis during the course of the working day.

The apprenticeship system in the workshop provided me with an opportunity to learn something new with a conscious awareness of Vygotsky's theory of learning (Vygotsky 1962, 1978, 1981; Wertsch, 1979, 1984; Wertsch, Minick, & Arns, 1984). Vygotsky's theory emphasizes the social nature of people's learning and argues that "the individual response emerges from the form of collective life" (Vygotsky, 1981, p. 165). In the workshop, under the tutelage of more experienced carpenters, I benefited from the collective wisdom of the group and learned to mathematize like a carpenter. Describing my learning experiences as an apprentice provided a way to talk about the mathematizing that the carpenters taught me using the tools, symbols, and metaphors of their culture. This rationale provided me some protection from the danger of being confined to describing only those aspects of the carpenters' mathematizing that resembled the academic mathematics in which I had been schooled. The results of this part of the study are reported in the form of an essay in Chapter 6.

In order to avoid the "everything is mathematics" danger, I used several frameworks through which to view the activities of the carpenters. Bishop's work (1988a, 1988b), in which he describes the six "environmental activities" of counting, measuring, locating, designing, playing, and explaining and from which he claims that all mathematical ideas can be generated, provided a starting point. However, finding activities in the workshop that fitted Bishop's categories was not enough to claim that such an activity was an example of mathematizing. There were certainly many activities that involved, for example, measuring

or counting but that I did not want to call mathematizing. Some woodworking operations had been routinized, and many were performed in a rote fashion, rather like a student who solves a calculus problem correctly by imitation and rote memory but without any understanding. I would not like to claim that mathematizing is occurring in either of these two cases.

Although Bishop's categories provided a framework for the kinds of activities that may be fruitful to explore, it seemed clear that *how* the activities were performed was of importance to my research. For this reason, I set up some rigorous criteria to apply to my observations in order to test whether or not I could argue that I had found evidence that the carpenters were generating and using mathematical ideas that would yield to description.

A basic requirement for the inclusion of an incident in the workshop as an example of mathematizing was that the carpenters, individually or in a group, were engaging in a cycle of action and reflection on a problematic. Other characteristics that I looked for in encounters were those examples that required the carpenter to explain something to me verbally or nonverbally, to offer some argument for verification, to appraise some work critically, or to act as a tutor providing "scaffolding" to assist an apprentice, "in the form of ideal modeling of a performance, hints, reminders, explanations, or even missing pieces of knowledge to assist the apprentice's task execution" (Gott, 1988, p. 99). The above requirements are not meant as an exhaustive list of the characteristics of mathematizing, but served the function of allowing me a method to sift through my experiences and observations and to concentrate on those that yielded rich results.

As an analytic tool, I referred to the work of Krutetskii (1976), who listed nine "basic characteristics of mathematical thought" (1976, p. 87) based on his studies with Russian school children (for a full listing, see Chapter 5). An analysis of the carpenters' problem-solving activities revealed characteristics of mathematical thinking that were similar to those described by Krutetskii. The results of this part of the study are reported as a series of episodes, followed by an analysis of those episodes, in Chapter 5.

Organization of the Monograph

The monograph is organized in the following way: Chapter 2 contains reviews and critiques of the relevant literature. Chapter 3 describes the "cast of characters"—the people who work in the workshop, as well as a brief history of the workshop. In Chapter 4, I set out the methodological foundations for the research and discuss the data collection methods.

Chapter 5 relates the story of the mathematizing revealed by the carpenters as they solved problems that arose spontaneously within their daily routine at the workshop. In Chapter 6, I tell my own story of how I learned to mathematize like a carpenter. In Chapter 7, some conclusions and suggestions for further investigations are discussed.

Chapter 2

CONCEPTUALIZING THE RESEARCH PROBLEM

Introduction

This chapter reviews in detail the literature introduced in the overview in Chapter 1. As the literature is examined and critiqued, a broader conceptualization of the present research problem will emerge. The chapter ends with a formal statement of the problem and a description of the goals of this research project.

Culture and Cognition

Lave (1988) argues that our thinking on cognition has been shaped by psychologists and our thinking on culture has been shaped by anthropologists. As a result of a sort of division of labor between the two disciplines, there has been a lack of meaningful discourse between them. Following the theory of developmental cognitive psychology developed by Piaget, research and theory on cognitive development in American psychology have been dominated by an examination of the individual's development, without much consideration of societal and cultural influences. Culture has often been treated as something to be factored out in a laboratory setting, rather than as an integral part of cognition (for exceptions to this approach, see Bronfenbrenner, 1979).

In the cross-cultural work done by cognitive psychologists, particularly that done within the Piagetian framework, the lack of attention to the complexities of culture has led to certain problems. Cole and Scribner (1974, p. 173) list the following problems that pervade cognitive research in cross-cultural settings:

1. There is a great readiness to assume that particular kinds of tests or experimental situations are diagnostic of particular cognitive capacities or processes.
2. Psychological processes are treated as "entities" that a person "has" or "does not have" as a property of that person, independent of the problem situation. They are also considered to operate independently of each other.

3. Closely related to (1) and (2) is a readiness to believe that poor performance on a particular test is reflective of a deficiency in, or lack of, "the" process that the test is said to measure.
4. Evidence from other disciplines (especially anthropology and linguistics) is usually not taken into account in making inferences about the cognitive processes that a given cultural group has or uses.
5. The complexity of the cultural groups and institutions studied is very often grossly oversimplified.

One unfortunate result of the points above is that much of the ethnographic work done by anthropologists and cross-cultural psychologists has been based on the concept of "cultural deprivation" (Keddie, 1973). For example, Bishop (1988a, p. 21) describes the seminal work, *The New Mathematics and an Old Culture*, by American researchers Gay and Cole (1967), as "clearly a book written by foreigners coming from a culture that sees itself to be superior to the Kpelle culture in some way."

Lave's (1988) stated aim is to develop a more adequate model of cognition in a cultural context. To this end, Lave recommends the study of cognitive activity in everyday settings. She conceives of a "social anthropology of cognition" (1988, p. 1) as a theory of practice that challenges the conventional assumptions about the impact of schooling on everyday practice. Her research in everyday settings (for example, in a grocery store) led her to reject the theory of learning transfer, since most of the grocery shoppers interviewed in the study were able to perform innovative and flexible mental calculations in the store when deciding which product to purchase but did not perform well on pencil-and-paper tests consisting of "school" versions of similar problems.

Lave argues that cognition cannot be seen as being separate from the social world, the people acting, the settings of their activity, and their actions. She attempts to develop a "theory of everyday activity," where the term "everyday" does not have its customary place in the "unsung category of humble domestic activities" (1988, p. 14). Rather, it refers

simply to the ordinary daily, weekly, or monthly cycles of humans, like the activities of a teacher and her pupils in the classroom, the work of a scientist in her laboratory, a person shopping for groceries in the supermarket, or a group of carpenters working. Lave (1988, p. 1) argues strongly against separating the individual from collective, cultural aspects of cognition, asserting that "'cognition' observed in everyday practice is distributed—stretched over, not divided among—mind, body, activity and culturally organized settings (which include other actors)." Culture and cognition are thus inextricably intertwined.

The work of Lave has strong links with the ideas of the Russian psychologist Vygotsky (1962, 1978, 1981), whose theory of cognitive development has raised the interest of educational and developmental psychologists in the U.S. during recent years. Vygotsky's theory emphasizes that cognitive functioning occurs first on the social level, between people, and that the child then internalizes this in individual development. Vygotsky does not claim simply that the mental processes of an individual develop within a social milieu; rather, according to Wertsch and Rogoff (1984),

> Vygotsky views individuals' mental processes as having specific organizational properties that reflect the social form from which they derive. The composition, structure and means of action are internalized from their social origins. This means variation in the organization of individual psychological functioning. For example, a child who has participated in joint problem solving will use the same task representation that proved effective in group problem solving when solving such a problem independently. (p. 2)

Vygotsky (1981) argues that the main focus of psychology should be "to show how the individual response emerges from the form of collective life" (p. 165). In contrast with the psychological investigations of Piaget, who measured the level of mental development of a child as being indicated by the standardized problems that the child was able to solve alone, Vygotsky (1962) studied the results of allowing children to work in collaboration with an adult on problem solutions. Using this approach, he discovered that one of two eight-year-old boys who worked on problems "in cooperation" was able to solve problems designed for twelve-year-olds, while the other could not go beyond problems intended for nine-year-olds. He introduced the term "the zone of proximal development" (1962, p. 103), which he

defined as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). In the example described, the first child had a zone of four and the second had a zone of one. Vygotsky (1962) claimed that the child with the larger zone of proximal development would fare better in school. Through the introduction to the child of the use of society's tools and techniques and their practice in the child's zone of proximal development with more experienced members of society, individual development is guided by the social world (Wertsch & Rogoff, 1984).

The Vygotskian school considers "activity" as the basic unit of study in psychology. The notion of activity focuses on the socioculturally defined context in which human functioning occurs and refers to an actual, identifiable activity as opposed to a generic notion of human activity (Wertsch et al., 1984). Some of the activities mentioned by Vygotskian psychologists are play, instruction or formal education, and work. Wertsch et al. (1984, p. 155) discuss the difference between the activity setting of formal schooling, where "learning for learning's sake" is valued and errors are expected and sometimes even encouraged, and the activity setting of a "work or labor activity," where error-free task performance is of overall importance. In the work context, the activity setting is structured so that learning may occur in a form similar to apprenticeship, where the novice's performance is carefully monitored until it is faultless. Vygotsky's theory is of particular interest for the present study in the carpentry workshop, where learning occurs in a system of apprenticeship.

The idea of "scaffolding," a process whereby a child learning to master a problem is given support by an adult, was developed by Wood, Bruner, and Ross (1976) and is closely related to the concept of the zone of proximal development. Rogoff and Gardner (1984) use the notions of "scaffolding" and the "zone of proximal development" to explore cognitive development in a study of mothers assisting their children to prepare for a memory test on

the categorization of objects. They describe the assistance as integrating both *explanation* (when the adult talks and the child listens) and *demonstration* (when the adult carries out the task and the child watches) with the *participation of the child* in the task. Rogoff and Gardner (1984) characterize instruction that uses scaffolding as

> careful guidance and graduated participation in the task . . . (which functions) as a deliberate but tacit process which the participants construct in the course of communication. They proceed opportunistically in the transfer of information and skills in a purposeful, flexible way, making use of pragmatic aspects in the context to develop means of instruction. Instruction can be conceived of as a complex, tacit process developed in the particular problem situation, rather than as an explicit recipe for problem solution that is available out of the context of action. (pp. 102–103)

This thorough characterization of the teaching and learning that occurs in the model of apprenticeship is extremely valuable. The integration of explanation, demonstration, and participation in the model is an important point that is *not* made explicit in other discussions of the wider use of the "apprenticeship" model of teaching and learning, such as the work by Brown, Collins, and Duguid (1989).

Brown et al. (1989), drawing on the research into cognition in everyday activity, propose the ideas of "situated cognition"[4] and of "cognitive apprenticeship" as an alternative to conventional school practices, arguing that "knowledge is situated, being in part a product of the activity, context and culture in which it is developed and used" (p. 32). Brown et al. suggest that students be "enculturated" into "authentic" practices through activity and social interaction in a way that is similar to craft apprenticeship, but what precisely should be taught to achieve enculturation is not clearly stated. The classroom practices of two exemplary mathematics teachers are described and are used to derive three "procedures that are characteristic of cognitive apprenticeship" (Brown et al., 1989, p. 38): namely, beginning with a task embedded in familiar activity, stressing that heuristics are not absolute, and allowing students to generate their own solution paths. Although I agree that the apprenticeship model has characteristics that may be useful to try to incorporate into

[4]The idea of "situated cognition" generated both excitement and skepticism in education circles. For critiques of the article, see Palincsar (1989) and Wineburg (1989).

classroom instruction, I do not think that the three characteristics described by Brown et al. can be said to be derived from the traditional apprenticeship model. Social interaction and collaborative learning are added as two more important features to be fostered in classrooms. These features *are* characteristic of apprenticeship learning and have been shown to be fruitful in the classroom. However, it is not enough to simply describe some of the seemingly more desirable attributes in learning situations and to give this collection the new label "cognitive apprenticeship." For the apprenticeship model to be seriously considered as an alternative to conventional classroom practices, there will have to be much deeper analyses of what it means to be an apprentice, what it means to be a "master/mistress" practitioner, and of the dynamic relationship that develops between novice and expert in the learning and teaching process, such as in the work on the "reflective practitioner" by Schon (1983, 1987). I intend in the present study to draw upon my experiences as a carpentry apprentice to comment on how I believe some of the characteristics of such a teaching and learning situation could be used in the mathematics classroom.

Knowing-in-Action and Reflection-in-Action

Schon (1983, 1987), in his proposal to redesign professional education for "reflective practice," has written extensively about his research into "the experience of learning by doing and the artistry of good coaching" (1987, p. 17). Schon (1987) proposes that the university-based professional schools should learn from the many traditions of education for practice, such as studios of art and design, conservatories of music and dance, athletics coaching, and apprenticeships in the crafts, all of which emphasize coaching and learning by doing. He draws on the work of Dewey to support his argument that students can be *coached* rather than *taught* what they need to know. Dewey (1974) stated:

> He has to see on his own behalf and in his own way the relations between means and methods employed and results achieved. Nobody else can see for him and he cannot see just by being "told," although the right kind of telling may guide his seeing and thus help him to see what he needs to see. (p. 151)

Many of our actions are intuitive and spontaneous and reveal a tacit knowledge that is difficult to describe explicitly. Schon (1983, p. 49), in describing the behavior of successful practicing professionals, claims that "our knowing is *in* our action. . . . The workaday life of the professional depends on tacit knowing-in-action." Some examples of this knowledge that is implicit in action include judgements of distance in golf or ball-throwing, the use of tools, or the recognition and correction of a "poor fit" in design (Schon, 1983). We often cannot describe the rules that govern these actions, although of course we can reflect on our actions both while we are acting (reflection-in-action), and after the fact. It is sometimes possible, by observing and by reflecting upon our actions, to describe the tacit knowledge that is implied by our actions. Schon (1987) states that—

> Know*ing* suggests the dynamic quality of knowing-in-action, which, when we describe it, we convert to know*ledge*-in-action. (p. 26)
>
> Whatever language we may employ, however, our descriptions of knowing-in-action are always *constructions*. They are always attempts to put into explicit, symbolic form, a kind of intelligence that begins by being tacit and spontaneous. (p. 25)

I hypothesize that the intuitive actions of the carpenters reveal a tacit knowledge of mathematics that is difficult for them, or for an observer, to describe explicitly. I shall use the word "mathematizing" to suggest the dynamic quality of this knowing-in-action. By observing their actions and performing the actions myself as an apprentice, by encouraging the carpenters to reflect upon their actions, by reflecting upon my own actions, and by reflecting upon their reflections, I intend to describe the mathematizing implicit in the actions related to carpentry.

The Epistemology of Constructivism

Piaget's work teaches that the essential way of knowing the world is not directly through our senses, but primarily through our actions. Action is understood as being all behavior by which we cause a change in the world around us or by which we change our own situation in relation to the world (Sinclair, 1987). Such behavior transforms the relationship between the knower and the known and sometimes results in the construction

of new knowledge, awareness, and meaning. Newly constructed action schemes remain viable until wider experience raises novel questions and creates situations in which the action schemes are inadequate and require further modification.

As pointed out by Confrey (1991), constructivists base their definition of the construction of knowledge on Piaget's principle of intellectual adaptation. Intellectual adaptation is an exchange between a person and his or her environment and involves assimilation and accommodation. Assimilation occurs when the cognitive system deals with environmental events in terms of its existing structures, and accommodation occurs when the cognitive system modifies itself as a result of environmental demands (Ginsburg & Opper, 1969). Children are always active, attempting to understand things, to structure their experience, and to bring coherence and stability to their worlds. Through assimilation and accommodation, they endeavor to reach an active harmony with the environment, which Piaget calls "equilibrium" (Ginsburg & Opper, 1969, p. 172). Von Glasersfeld (1982) claims that Piaget's concepts of action scheme, assimilation, and accommodation have been widely misunderstood and have been misinterpreted so as to conform to the traditional beliefs of realists. Von Glasersfeld (1982, 1984) puts forward a view of "radical constructivism" (which explicitly rejects the notion that knowledge must match an absolute reality) as the epistemological view he considers to be most in harmony with Piaget's work.

It is "radical constructivism" that provides a framework for allowing multiple and competing representations of knowledge to co-exist. Von Glaserfeld (1984) describes a theory of knowledge "in which knowledge does not reflect an 'objective' ontological reality, but exclusively an ordering and organization of the world constituted by our experience" (p. 24). Whereas realists expect that scientific and everyday knowledge provide a true picture of the real world, independent of any knower's experience, radical constructivists believe that the human activity of knowing can lead only to a conjectural interpretation of the world; thus we construct our own realities (von Foerster, 1985; von Glaserfeld, 1984). Instead of the traditional conception of a "match" between knowledge

and reality, von Glasersfeld conceives of a "fit" between knowledge and reality. He uses the metaphor of a key, or more than one key, able to "fit" the same lock, to illustrate the human endeavor of constructing our own keys to fit the locks of our experiential world, the testing ground for our cognitive structures (von Glasersfeld, 1984). Our knowledge remains useful and viable so long as it fits the demands and expectations of our experiences. Knowledge that collapses under the demands of our experiential world "becomes questionable, unreliable, useless, and is eventually devaluated as superstition" (von Glasersfeld, 1984, p. 24).

Constructivism is frequently criticized as being susceptible to relativism; skeptics claim that since there is no "certain knowledge" to appeal to, then everybody's knowledge must be equally valid (Confrey, 1985, p. 5). However, such criticism ignores the fact that the "role of others" in the construction process is a fundamental aspect of constructivism (Confrey, 1985, p. 5). Through communication with others and by interpreting the acts of others, we receive feedback about our constructions. Through a process of argumentation and negotiation within communities, people try to reach some consensus of meaning about their personal constructs (Cobb, 1986a, 1986b; Confrey, 1985)

Cobb (1985), Confrey (1984a, 1984b, 1985, 1987, 1991), Kamii and DeClark (1984), Labinowicz (1985), Steffe (1986, 1988), and Steffe, Cobb, and von Glasersfeld (1988) have applied the epistemology of constructivism to mathematics education. The construction process of mathematical knowledge described by Confrey (1987, 1991) occurs in part as the result of repeated cycles of action and reflection on a problematic (see Figure 2). Confrey (1991) gives a detailed explanation of her model of the constructivist cycle. I shall paraphrase her explanation in the paragraph that follows.

For the constructivist, a problem does not have an independent existence; it is defined solely in relation to its solver. The problem is seen as a "roadblock" to where an individual wants to be. The term "problematic" is used to refer to the individual's "roadblock." When an individual acknowledges his or her problematic, equilibrium has

been disrupted, and the tendency is to work towards coherence and stability. In this way, the problematic acts as a "call to action." The "call to action" serves as an invitation both to the problem solver and to the researcher/teacher: while the problem solver is invited to engage in action and reflection, the researcher/teacher is invited to seek an understanding of the problematic being solved, the actions chosen, and the nature of the reflections. The actions taken to resolve a problematic may involve sensory-motor activity and/or cognitive operations. Reflection serves to provide feedback on the results of the action. Has the problematic been resolved? Does the problematic need to be modified? What is an appropriate choice for the next action (if necessary) to begin a new cycle? When an action is seen to be repeatedly successful in moving toward resolution of the problematic, that action may be objectified, named, or turned into a tool and used for future actions.

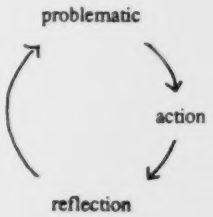

Figure 2. The constructivist cycle.

Thus mathematical knowledge is not transferred as a ready-made commodity from one person to another; rather, people build their knowledge on the basis of their experience (Steffe, 1986). According to Labinowicz (1985, p. 5), arithmetical knowledge is "constructed actively by the child in the process of adapting to his environment." Cobb (1986b) states that in the constructivist model of instruction, classrooms are viewed as places where teachers and students negotiate as they attempt to construct shared meanings. Learning is thus a process of communication in which the teacher listens to the children, assumes that the child's behavior is rational, and tries to identify and understand the child's goals. Confrey (1991) describes two basic guiding commitments for the constructivist

researcher/teacher; first, that the meaning of the mathematical insights constructed by individuals "lies within the framework of the individual's experience," and second, that the individual's inventions and explanations "have legitimate epistemological content."

The first assumption of the constructivist view of mathematics is described as follows (Confrey, 1991):

> Constructivists view mathematics as a human creation, evolving within cultural contexts. They seek out the multiplicity of meanings, across disciplines, cultures, historical treatments, and applications. They assume that through the activities of reflection and of communication and negotiation of meaning, human beings construct mathematical concepts which allow them to structure experience and to solve problems. Thus, mathematics is assumed to contain more than its definitions, theorems and proofs and its logical relationships . . . included in it are its forms of representation, its evolution of problems and its methods of proof and standards of evidence.

As pointed out earlier, Piaget's theory of cognitive development was built as an individualistic model. Social and cultural aspects of development were explored only so far as they affected the individual. Since constructivism developed from the ideas of Piaget, it is not surprising to find the collective, societal elements underrepresented in the epistemological framework of constructivism, especially in relation to peers. However, recent work in constructivism is moving toward according the social and cultural aspects of knowledge construction a much higher priority than previously (Cobb, 1988; Newman, Griffin, & Cole, 1989; Yackel, Cobb, & Wood, 1990).

Constructivism is seen as a vital part of the conceptualization of the present research. The simple model of action and reflection on a problematic in the constructivist cycle has practical uses during fieldwork designed to recognize and describe the mathematical practices of a community of people, since problem solving is one of the occasions during which their mathematizing is demonstrated. The identification of, and engagement with, a problematic serves as a signal to the researcher to take careful note of all the interactions taking place. Since, as Schon (1983, 1987) claims, tacit knowledge resides in actions, it is important to notice all the details of these actions and to reflect on

them later. It is in reflecting on the actions and the reflections observed that the possibility of describing the tacit knowledge develops.

The commitment of constructivist researchers to the belief that the inventions and sometimes seemingly obscure and clumsy attempts of all people represents legitimate mathematical knowledge, however tentative and partial, lessens the tendency of the researcher to "switch off" when the arguments seem irrelevant and implausible. There is a greater tendency to try to follow and understand the explanation and very often to discover that it is one's own way of hearing that needs modification. As one listens, one learns to appreciate the power of constructing multiple representations of single concepts. Traditional school mathematics can more easily be seen as one possible conception of mathematics and not an international benchmark according to which all the mathematical endeavors of humankind must be judged right or wrong.

What Is Culture and What Constitutes a Cultural Group?

Most anthropologists today agree with the semiotic concept of culture espoused by Geertz (1973, p. 5), who believes that "man is an animal suspended in webs of significance he himself has spun." Geertz regards culture as those webs. It is through cultural patterns and ordered groups of significant symbols that people make sense of the events in their lives. These symbol systems are constructed historically and are maintained through social interactions among individuals. The system of symbols comprising a culture can be interpreted or "translated" (R. Ascher, personal communication, November 1988) to some extent by an observer. This is the main endeavor of anthropology, which regards culture as a context in which social events, behaviors, processes, and institutions can be described by an ethnography. In the study of culture, symbolic acts are "diagnosed" to "ferret out the unapparent import of things" in the process of analysis of social discourse, much like the way a physician would diagnose symptoms in a patient (Geertz, 1973).

Giroux (1981) criticizes the work of Geertz for contributing little to the understanding of how political power structures the socioeconomic classes, institutions, and

social practices within a society. He describes the concept of culture as being "stripped of its political dimensions," "reduced to an anthropological or sociological object of study," and "an apology for the status quo" (Giroux, 1981, p. 26). He calls for an acknowledgement that any distinction between culture and power is false. Culture could thus be redefined in terms of its relationship to the dominant social formations and power relations in society. This "politicized" concept of culture would include the relationship between ideology and the socioeconomic system. Giroux (1981) describes a "politicized" notion of culture more fully thus:

> Culture, in this sense, would be defined not simply as lived experiences functioning within the context of historically located structures and social formations, but as "lived antagonistic relations" situated within a complex of socio-political institutions and social forms that limit as well as enable human actions. (p. 26)

In countries where the political and economic dominance of one group over all others has been entrenched for many years, the concepts of power and conflict are central to a discussion of culture. In South Africa, one cannot discuss educational issues and learning opportunities without recognizing and acknowledging the existence of a myriad of unwritten constraints and social taboos rooted in racism in addition to the overt forms of discrimination legislated by apartheid. Although the present study does not follow the ideas of Giroux fully, it is recognized that culture is more than the expression of that which grows out of the experiences of people in their daily social and working roles. Working communities share a culture defined not only by the system of symbols developed historically through the practice of their craft or trade; political and socioeconomic factors also play a significant part in shaping the culture of a group. Thus it is claimed that a group of carpenters such as the one selected for this study constitutes a cultural group, although there may be a diversity of languages and religions among members. The notion of culture used in the present study supports D'Ambrosio's (1985) broad conceptualization of what constitutes a cultural group.

For the purposes of the present research, the culture in the workshop is seen to include the "antagonistic relations" that exist at some level between the artisans who wish

to protect their turf (as well as their socioeconomic status) and the workers who struggle for opportunities to learn and to improve their social and economic status. The ability of all the employees to shape their lives within social, political, and economic constraints is also recognized as part of the culture of the workshop.

The Cultural, Social, and Political Aspects of Mathematics and Mathematics Education

Bishop's statement that "mathematics is a pan-cultural phenomenon" (1988a, p. 56) is supported by the existence of Chinese mathematics, Greek mathematics, Roman mathematics, African mathematics, Islamic mathematics, Indian mathematics, and Neolithic mathematics. By referring to this long list, Bishop concludes that different cultures produce different mathematics. Further examples of mathematics that differs from that usually found in Western textbooks can be found in the work by Zaslavzky (1973), Brenner (1985a, 1985b), and Gerdes (1985, 1986, 1988a, 1988b) in Africa; Bishop (1979), Lancy (1983), and Lean (1986) in Papua New Guinea; and Pinxten, van Dooren, and Harvey (1983) with the Navajo. This cross-cultural literature suggests that all cultures develop their own forms of mathematics depending on the demands of their particular environments and the goals and purposes of the people.

Bishop (1988a) explains that throughout history, culture contact and culture conflict in the form of trade, religion, education, and warfare have done much to eliminate cultural diversity in the area of mathematics and have also contributed to the growth of "Mathematics" as an internationalized discipline. Bishop (1988a) emphasizes that Mathematical ideas grow as a result of developments taking place both within cultures and between cultures. The label, "Mathematics," (which usually means Western academic mathematics) is generally used as if all the ideas referred to by the label were developed by Western civilizations. This is not the case, as an honest interpretation of historical records will reveal (Bishop, 1988a; Gerdes, 1985; Joseph, 1987). It is important that students all over the world become aware of the fact that, with few exceptions, a strongly Eurocentric

view of mathematics has permeated their schooling. The histories of societies outside of the European tradition and their contributions to mathematics have been misrepresented, and the "imperialist/racist ideology of dominance" that underpins the widespread Eurocentric view prevails despite convincing evidence to the contrary (Joseph, 1987, p. 16). What we call "Mathematics" is made up of contributions from many cultures, many of them non-Western (Bishop, 1988a; D'Ambrosio, 1987; Gerdes, 1985).

Bishop (1988a) refers to the many different concepts and definitions of culture that have been put forward by cultural anthropologists. He also discusses the work of the mathematician Kline (1980), which focuses on the culture of mathematics, and the work of mathematician/historian Wilder (1981), which focuses on the cultural group of mathematicians. Bishop rejects both of these approaches as models because of their exclusion of nonmathematicians. While his work refers mainly to cultural groups that are national groups or language groups, he does not give a clear statement as to what concepts of culture and cultural group he, as an educator, chooses to use.

D'Ambrosio (1985) suggests that it may be fruitful to explore the mathematical activities of smaller communities with common purposes and interests. His conceptualization of the kind of cultural group that generates its own forms of mathematics does not remain restricted to national groups but is much more encompassing. D'Ambrosio (1987) states:

> our conception of "Ethno" encompasses all the ingredients that make up the cultural identity of a group: language, codes, values, jargon, beliefs, food and dress habits, physical traits. . . . Cultural groups, children of a certain age range in a neighborhood, farmers cultivating wheat, engineers in car factories, and so on, all have their own patterns of behavior, codes, symbols, modes of reasoning, ways of measuring, of classifying, of mathematizing. (pp. 4–5)

While both Bishop and D'Ambrosio claim that different cultural groups generate different forms of mathematics, they emphasize that it is through a very similar range of human activities that these different forms result (Bishop, 1988a; D'Ambrosio, 1985, 1987). The identity of the mathematics depends, for example, on the environment, the context, the focus of interest, the motivations, the forms of communication, the goals, and the purposes

of each group, and there will be many differences in these areas. However, there are strong similarities between cultures as well.

Bishop describes six "universal" activities, which, he argues, form the foundations for the development of mathematics and which are extremely widespread, if not universally practiced in every culture. *Counting* and *measuring* are both concerned with ideas relating to number. The significant feature of *counting* is its discreteness in the association of objects with numbers, in contrast to the continuous nature of *measuring*. *Measuring* involves comparing, ordering, and quantifying characteristics that are important. The activities of *locating* and *designing* fall into the category of spatial structuring, which has been of great significance in the development of mathematical ideas. In *locating* activities, the emphasis is on the topographical and cartographical features of the environment—how space is conceptualized and how people and objects are positioned in the spatial environment. *Designing* refers to the conceptualization of objects and artifacts that are manufactured for use in our homes and leads to the idea of "shape." Two activities that link us to our social environment are *playing* and *explaining*. *Playing* is concerned with social procedures and rules of performance and also stimulates the "as if" feature of imagined and hypothetical behavior. *Explaining* refers to the various cognitive aspects of enquiring into and conceptualizing the environment and of sharing those conceptualizations. *Explanations* establish meaningful connections between different phenomena in response to the question "Why?"

These categories are useful for directing the researcher's attention to those human activities that are likely to be fruitful areas of further study if one is interested in documenting instances of mathematizing within a community of people. Members of the community who identify problematics that involve these categories of activity, and who engage in the constructivist cycle, are likely to construct or use some form of mathematical knowledge to reach solutions.

Fasheh (1982) points out that a prevalent misconception in the teaching of mathematics has been, and still is, the belief that mathematics can successfully be taught without relating it to culture or to the individual student. Stigler and Baranes (1988, p. 259) identify three primary points at which culture influences mathematics learning. These points are (a) cultural tools, (b) cultural practices, and (c) cultural institutions.

Cultures provide their members with a variety of tools that, through processes of internalization, become a part of the individual's range of mental representations. These representations can be used for thinking about mathematics and for solving mathematical problems. Mathematical thinking is shaped by tools like language as well as concrete cultural artifacts like the abacus, widely used in Taiwan, Japan, and other Asian countries, and like the compass rose used by medieval navigators. Both the abacus and the compass rose provided a "representational structure that enabled potentially complex calculations to be easily performed mentally" (Stigler & Baranes, 1988).

Cultural practices, such as the navigational traditions of illiterate Micronesian islanders, were shown to require such tasks as estimating and calculating distances and angular measure, while the fish marketing strategies of women in Ghana made use of the concept of probabilities (Stigler & Baranes, 1988). Other examples of cultural practices are those given by Gerdes, such as the Mozambican practice of fish drying, which depends upon an understanding of the circle concept (1985, p. 17); of house building, which is based on an understanding of the properties of a rectangle (1985, p. 17); and of basket weaving, which uses the results of the Pythagorean theorem (1986, p. 12). Stigler and Baranes (1988) suggest that cultural practices are structured in a way so as to eliminate the need for repeated complex calculations. Tools and rules are developed so as to optimize efficiency and ease of performance.

The institution of schooling is surrounded by a cultural context that transmits both intended and unintended beliefs, values, and practices to its participants. Some of the unintended beliefs are that mathematics problems can always be solved in less than 10

minutes and that there will always be an answer to a problem that contains numbers, regardless of the meaning of the problem; for example, three out of four elementary school students gave a numerical answer to the problem, "There are 26 sheep and 10 goats on a ship. How old is the captain?" (Stigler & Baranes, 1988, p. 288). Bishop (1985, p. 26) reminds us that although a classroom is part of the institution of the school, each classroom group is a unique combination of people, with "its own identity, its own atmosphere, its own significant events, its own pleasures and its own crises. . . . It has its own history created by, shared between and remembered by the people in the group."

References to the social aspects of mathematics can also be found in the work of some mathematicians and philosophers, who offer a challenge to the traditional conceptions of mathematics. The works of Lakatos (1976) and Tymoczko (1980, 1986) are seen as particularly relevant for the present study. Lakatos (1976) regards mathematics as intrinsically speculative and fallible, growing, like the Popperian view of science, as a pattern of conjectures, criticism, counterexamples, refinement of theories, refutation, and further refinement. He does not reject formal mathematics but rejects the idea that mathematics grows in the deductive pattern of formalization. It is claimed that the logic of mathematical discovery is naturally deductive only if one restricts mathematics to formal axiomatic systems where progress can occur only in measured logical steps. This conception of mathematics is falsely implied by formal deductive proofs, which are presented in a way that conceals the primitive conjecture and disguises the criticism and refutations. In the words of Lakatos (1976):

> In deductivist style, all propositions are true and all inferences valid. Mathematics is presented as an ever-increasing set of eternal, immutable truths. Counterexamples, refutations, criticism cannot possibly enter. An authoritarian air is secured for the subject by beginning with disguising monster-barring and proof-generated definitions and with the fully-fledged theorem, and by suppressing the primitive conjecture, the refutations, and the criticism of the proof. Deductivist style hides the struggle, hides the adventure. The whole story vanishes, the successive tentative formulations of the theorem in the course of the proof-procedure are doomed to oblivion while the end result is exalted to sacred infallibility. (p. 142)

The book *Proofs and Refutations* (Lakatos, 1976), most of which takes the form of a discussion between a teacher and his students, is a "rational reconstruction" of the development of the proof for the Descartes-Euler conjecture about the relationship between the number of vertices (V), edges (E), and faces (F) of polyhedra, namely that $V - E + F = 2$. Lakatos gives a vivid illustration of how mathematical knowledge is constructed from the interactions of a group of mathematicians. The students represent various mathematicians who tried to prove Euler's theorem from the mid-18th century throughout the 19th century. The social exchanges, arguments, disagreements, attempts to convince, attempts to find a hole in another's argument, criticisms, and elaborations of another's work cannot take place in the absence of a community, although this point is not explicitly made. It seems clear that Lakatos supported the idea that mathematical knowledge is socially constructed and is not individually discovered, but it is doubtful that his concept of communities included people other than professional mathematicians.

While Lakatos made no specific references to the role of community in the creation of mathematical knowledge, Tymoczko (1986) centers his argument on the social aspects of knowledge construction. Tymoczko states that the prime concern of the philosophy of mathematics should be to argue for the role of the community of mathematicians instead of focusing on one individual, isolated mathematician. He argues that mathematics is a human activity, and he urges for recognition that mathematics is public knowledge and that it is communication among mathematicians in their community that grounds mathematical knowledge (Tymoczko, 1980). However, Tymoczko neglects the point of view of the student, who is portrayed as the passive recipient of the accumulated wisdom of experienced members of the mathematics community. The student is not considered to be a participant in the creation of knowledge and in the solution of problems but as an initiate, to be molded in the image of the current members of the elite community of professional mathematicians. As with Lakatos, Tymoczko's view of mathematics does not seem to encompass the creators and users in the world outside of the ivory tower: the children,

students, and "ordinary" people who also create and use mathematics to explain, convince, and communicate.

The mathematics educator Steffe (1986, 1988) and his colleagues (Steffe et al., 1988) have developed a model of how young children construct the number sequence using their early counting schemes. There is also evidence that children create different types of units and base their conceptualization of multiplication on those units. Steffe describes this as part of the "mathematics of children" and voices his explicit intention that "school mathematics will at least include the mathematics of children" (1986, p. 138).

Apart from the children and students who have been left out as genuine makers and users of mathematics, other groups have also consistently been denied the status of fully fledged participants. Harris (1987) points out that in most histories of mathematics, women mathematicians are scarcely mentioned. In fact, throughout Western history the works of women mathematicians have been "ignored, robbed of credit and forgotten" (Alic, 1986).

Harris (1988b) contrasts the low achievement of girls in school mathematics with their capacity to do complicated needlework. In her research with women who make and use textiles at home and in factories, Harris (1987) points out the similarities between the problem of shaping the heel of a knitted sock and the problem of enclosing a right-angled cylindrical pipe in a factory with heat-insulating material. In essence, the mathematical problems to be solved are the same; however, the industrial problem would typically be considered to require mathematical thinking, and the sock problem would not. Similarly, the complicated symmetrical designs that illiterate women weavers construct in their rugs are not considered as evidence of any form of intellectual thought. However, Harris (1988a) points out that in cultures such as in the Gambia, where men do the weaving, the educational aspect of weaving is claimed "under the equation weaving = knowledge = power." Harris (1988b) concludes that the problem lies in who defines what mathematics is and in who defines what the standards are. In a male-dominated society, women's

mathematics is not acknowledged. Fasheh (1988) made similar observations about his mother's mathematics:

> My illiterate mother routinely took rectangles of fabric and, with few measurements and no patterns, cut them and turned them into beautiful, perfectly fitted clothing for people. . . . The math she was using was beyond my comprehension. . . . Seeing my mother's math in context helped me see my math in context, the context of power. What kept her craft from being fully a praxis and limited her empowerment was a social context which discredited her as a woman and uneducated and paid her extremely poorly for her work. (p. 2)

Gerdes (1988b) explains how racist and colonial ideology negated the capacity of Mozambicans to do mathematics. The indigenous wisdom and mathematical knowledge of the colonized were ignored or ridiculed and have left a legacy of self-doubt. The aftermath of years of repressive education systems further exacerbate the problems of mathematics education. Gerdes (1985) states:

> During the years of Portuguese domination, mathematics was taught, in the interests of colonial capitalism, only to a small minority of African children. And those Mozambicans were taught mathematics to be able to calculate better the hut tax to be paid and the compulsory quota of cotton every family had to produce. They were taught mathematics to be more lucrative "boss-boys" in South African mines. (p. 15)

In South Africa, the educational policies of the colonial era became more deeply entrenched with the rise of the racist industrial capitalist society. As stated by Kallaway (1984):

> Above all, schools negated the common sense knowledge of the colonized by reinforcing the self-image of incompetence and ignorance for those who did not go to school or those who failed at school. This often also led to those who were schooled to despise their own culture and traditions in favour of those of the colonizer or at very best to become ambivalent about their links with the "traditional" past or social milieu. This process has been described as a "culture of silence"—where the colonial element in schooling is its attempt to silence, to negate the history of the indigene, to rationalize the irrational and (to) gain acceptance for structures which are oppressive. (p. 9)

In the segregated South African schools, black students are often taught mathematics in a particularly authoritarian manner as a set of formulas and rules to be memorized and a set of problems (which have no connection to the students' lives) to be solved (Dostal & Vergnani, 1984). Many students believe that there is only one "correct" answer and that their task is to find this answer and to ratify its truth with the authority

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vested in the teacher or the authority vested in the answers printed in a list at the back of the textbook. Approaches to textbooks, curricula, and instruction in South Africa have been imported from Europe and the U.S.; there are no discussions of any African contributions to mathematics in texts. This all adds to the alienation of the students from the subject matter. Even when care has been taken to translate the text into the mother tongue of the students, problems remain (Berry, 1985).

During recent years, students, parents, and teachers from black communities have rejected state-controlled education and have demanded schooling that provides opportunities for people to pursue their own goals and to realize their social, economic, and political potential. "Education for Liberation" is a popular slogan among black South Africans, and there is community-based support for "People's Education." The work on "People's Mathematics" includes the basic mathematics and statistics required, for example, to analyze and criticize important issues in communities such as labor and rent disputes, newspaper reports, and differential government funding of education for different race groups. Thus, this work forms a valuable contribution to people's growing awareness of their own exploitation (Breen, 1986). The most vital issue is obviously not simply to point out injustices but to empower people to believe in their own abilities to mathematize. The destructive effects on black students of always seeing school mathematics as coming from somewhere outside of their own culture, as something a few hard-working and successful among them may "borrow" and use, but never really take ownership of, should not be overlooked.

Mellin-Olsen (1987) describes how the mathematics classroom can convert the passiveness, indifference, and destructiveness of some Norwegian students, who have rejected mathematics, into constructive activities, leading to inquiry, discovery, construction of new perspectives, and enhanced social responsibility. For example, when students in a large secondary school wanted to burn their textbooks because they contained "a lot of rubbish," Mellin-Olsen (1987) suggested, "You should collect some statistics. See

what it is in this book that makes sense to you. You should collect some data about it and report back to the publisher. Ask them why they publish such rubbish" (p. 200). In this way, he hoped to turn their negative energy towards activities that could generate self-directed learning.

In thinking about the issues of authority and conflict in mathematics education, I support the following statement made by Fasheh (1982):

> In short, I came to believe that the teaching of mathematics, like the teaching of any other subject in school, is a "political" activity. It either helps to create attitudes and intellectual models that will in their turn help students grow, develop, be critical, more aware and more involved, and thus more confident and able to go beyond the existing structures; or it produces students who are passive, rigid, timid and alienated. There seems to be no neutral point in between. (p. 7)

Research on the Creation and Use of Mathematics in Everyday Settings

As discussed in Chapter 1, the use of mathematics in everyday settings has been studied by two main groups of researchers. In the next two sections I shall examine and critique the work done in "everyday cognition" and in "ethnomathematics" and elaborate on how questions raised by these areas of research will be addressed by the present study.

Everyday Cognition

Lave's theoretical arguments have been discussed in a previous section. Lave and her colleagues have also produced a wealth of empirical evidence that highlights the relevance to mathematics education of studies about the mathematics used in everyday practice.

Early research with "schooled" and "unschooled" tailors and cloth merchants who had learned their professions via apprenticeship systems showed different results regarding the effects of schooling on the ability to do school-type arithmetic tasks that were similar to the arithmetic tasks performed with ease on the job. Lave (1977) and Reed and Lave (1979) studied the problem-solving skills of Liberian tailors. Tailoring was described as a skill that entails much quantitative computation, such as measuring, fitting, and calculating payments. Both schooled and unschooled master tailors were equally expert at these tasks. Lave gave "tailoring" and nontailoring, "school-type" tasks to the master tailors. All the

tailors performed equally well on the tailoring tasks, but the schooled tailors performed much better on the school-type tasks than did the nonschooled tailors. Lave interprets the ability of the nonschooled tailors to solve some of the school-type tasks as evidence that the apprenticeship training allows "general problem-solving principles" to develop (1977, p. 179). However, the evidence suggested that the schooled tailors had developed much more powerful generalized strategies for solving novel problems.

Contrary to these findings that traditional people found abstract manipulation of numbers difficult outside of familiar concrete contexts, Petitto's research (1979) with Dioula tailors and cloth merchants from the Ivory Coast showed that schooling had little or no effect on their ability to perform novel arithmetic problems that were analogous to the arithmetic computation skills developed during apprenticeships or in practice. Petitto (1979) investigated the arithmetic practices of the cloth merchants and of the tailors in their usual context, in unfamiliar contexts, and in abstract school-type problems. Petitto found her illiterate subjects had little difficulty in solving arithmetic problems presented in a purely numerical form, without context. General computational skills were organized into specific strategies that involved the numerical relationships themselves and did not depend on concrete contexts. Strategies learned in the context of practical activity were found to be transferable to directly analogous problems in novel settings. Petitto (1979, p. 197), however, pointed out that "the familiar and unfamiliar practical arithmetic problems may be more analogous to each other in our study than in Lave's" and also that the schooled tailors in Lave's study had considerably more years of schooling than did the schooled tailors in Petitto's study.

Schliemann (1984) studied problem solving among a group of professional carpenters and a group of carpentry apprentices with a range of educational backgrounds. The carpenters had from 0 to 5 years of formal schooling and had learned their professions while working as assistants to the owner of a workshop, usually their own father. The apprentices were attending a 3-year course of instruction in carpentry, as well as attending

formal school, and had at least 4 years of school mathematics. In the carpentry classes, great emphasis had been laid on measurement and on how to calculate area and volume. Schliemann reports that in the study, each of the carpenters and apprentices was given a clear, well-labelled diagram of a bed and asked to find out how much wood would be required to build five such beds.

Although most of the professional carpenters used suitable strategies to find a solution and displayed an understanding of the concept of volume, the apprentices' attempts were unsuccessful and the results obtained were absurd. Although they had received formal instruction on how to calculate volume, most apprentices either simply added together all the dimensions and gave a numerical answer, or added together all the lengths, all the widths, and all the thicknesses to give the dimensions of a huge block of wood. Their formal knowledge of the algorithm for calculating volume did not help them to solve the practical problem.[5]

Schliemann and Acioly's work (1989) with schooled and unschooled bookies who run a lottery game in Brazil showed that at work, regardless of the level of schooling, the bookies were extremely efficient. When asked to solve modified problems, schooling did not influence the use of specific algorithms for performing arithmetic operations, although the schooled bookies were better able to analyze and explain verbally the relationships and mathematical models involved in the lottery games.

Saxe (1988) investigated the mathematics of Brazilian child candy sellers. His findings show that sellers with little or no schooling develop, in their trade, arithmetic practices that differ from the arithmetic taught in schools. Peers who did not sell candy showed no evidence of similar knowledge. Furthermore, it was found that young school-going candy sellers used strategies analogous to those used in "currency arithmetic" to solve practice-related addition problems at school. For example, 28 + 26 = ? was reorganized as

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[5]Compare this situation to the discussion of the carpenters' understanding of volume in the present study. See episodes 15–19, Chapter 5.

(20 + 20) + (8 + 6) = ? This tendency faded with time, and older candy sellers used standard school-taught algorithms to solve school problems. Over time, however, candy sellers developed increasingly complex and powerful mathematics related to their trading activities, constructing concepts of ratio and continuing to construct problem-solving strategies that relied on the structure of the currency system.

Carraher et al. (1985) conducted research with child street-vendors in Recife, Brazil. Researchers posed as customers and asked the vendors to perform a variety of transactions on possible purchases, and later the children were asked to take part in a formal pencil-and-paper arithmetic test. As with Lave's grocery shoppers, results showed that problem solving in the market place was significantly more successful than problem solving on the formal test. There was evidence that the strategies brought to bear on daily trading problems were not the routines that the children had learned in school. Children relied on mental calculations, and the preferred strategy for multiplying seemed to be chaining successive additions. In other examples, as the addition became more difficult, the child decomposed a quantity into tens and units, to add 35 to 105, first adding 30, and then adding 5 later. In the formal test, the children tried, without success, to follow the problem-solving routines taught in school.

These research findings raise two important questions: first, how is mathematical knowledge developed from everyday experience, and second, how can this out-of-school knowledge be harnessed as a strength and used as a foundation from which to develop mathematics in the classroom?

Carraher and Carraher (1987) attempt to address the second question, arguing that the strategies used in the arithmetic that is learned outside of schools ("naive arithmetic") are based on the same principles as the arithmetic taught in school. For instance, the decomposition and repeated grouping methods used by the young street vendors are based on associativity, commutativity, and the distributive property of multiplication. Thus naive arithmetic can be extended into school arithmetic. However, as a result of the differences in

the situations in which "naive" and school arithmetic are practiced, problem solvers experience more success with "naive" arithmetic. In the market place, answers are evaluated for appropriateness and rejected if they do not make sense; at school, appeals to common sense are fruitless, since concepts are usually "stripped of situational components during the learning process" (Carraher & Carraher, 1987, p. 11). Schools must therefore find contexts that make concepts meaningful and useful in daily situations.

Brenner's (1985a, 1985b) work with Vai elementary students in mathematics classrooms gives an excellent example of how the Vai children combine two distinct arithmetic systems to master the school curriculum. Vai children from Liberia learn to count and perform a variety of arithmetic calculations in Vai as a natural part of learning their language and through participation in home and school activities. Simultaneously, they are engaged in learning arithmetic in English at school and are using algorithms similar to those taught in U.S. schools.

Vai arithmetic is characterized by combining strategies such as extensive counting, breaking problems into smaller units, converting multiplication into a series of addition steps, mental calculations, and memorized facts. Brenner observed that the children used a combination of Vai, school, and invented arithmetic algorithms during individual testing and classroom lessons. Those who used the widest range of methods were the most successful. Teachers were all members of the Vai group and were supportive of the children's styles. The pattern of teacher/student interactions in classrooms was culturally appropriate, and lessons were structured so that children could display their competence rather than their mistakes.

The research reviewed so far gives evidence that in most cases arithmetic that is developed in everyday settings is useful for solving novel problems; in some cases, even noncontextual, school-like problems were solved. There has also been considerable work done to explore the opposite question. Of what use is school mathematics in the solution of real-life practical problems encountered by educated adults in everyday contexts?

Lave et al. (1984) and Murtaugh (1985) found that arithmetic practices taught in schools were ignored by U.S. grocery shoppers in favor of methods such as approximation and estimation, which better suited the goals and purposes of the grocery shoppers. Unit price information was not generally calculated, nor was it used if provided by the store. Quantitative information, such as best-buy calculations, were used only after the options had been narrowed down to two or three. Initial selecting was based on brand, quantity, size, and storage considerations. The emphasis was thus on efficiency and time-saving. When best-buy calculations were performed, all shoppers, regardless of their level of schooling, were equally successful. However, the results of a pencil-and-paper test of arithmetic problems showed that the shoppers, who had carried out virtually error-free arithmetic in the supermarket, averaged only 59% on the arithmetic test. The number of years of schooling correlated highly with performance on the test but not with the frequency of calculations in the store. In everyday situations in the grocery store, school-taught arithmetic seemed rather useless.

In the study by de la Rocha (1985) with participants in a Weight Watchers dieting program, a program that requires precise measurement of foods, the emphasis was also on time saving and efficiency and not on employing school-learned arithmetic skills. De la Rocha found that the dieters invented strategies to simplify their measuring tasks in the kitchen. For instance, a measured amount of food was placed in a container with a decorative pattern and the levels noted relative to the pattern. The same container was then used repeatedly to eliminate the need for continual measuring.

Scribner (1984, 1985) reported that dairy workers developed efficient, labor-saving strategies based on physical characteristics like the layout of the dairy and the structure of product containers. For instance, instead of collecting items from shelves in the warehouse in the sequence laid out by the order forms, dairy workers organized and coordinated their activities in such a way as to minimize the distance to be traveled around the huge warehouse.

In the dairy warehouse, quarts of milk were packed 16 units to a case. On the order form, orders for 17 to 24 quarts were expressed as 1 case + 1 unit, up to 1 case + 8 units. Orders for 25 to 31 quarts were expressed as 2 cases – 7 units up to 2 cases – 1 unit. However, in making up milk orders, the workers did not adhere to the way in which orders were written on the order form but were observed to have large repertoires of solution strategies, depending on what configuration of cases was immediately available in the storeroom. In this way, mental energy was used in order to save the physical effort of moving extra cases. The workers were found to rely on visualization, rather than on arithmetic, to solve problems.

Examples of research of this kind are compelling because they raise fundamental questions about the usefulness of school-taught formal mathematics for people's lives. What is it about academic mathematics or about the way in which we teach mathematics in schools that leaves people unwilling or unable to apply that knowledge usefully in everyday settings? People are fully capable of devising innovative mathematical methods to solve the problematics and to accomplish the goals that they set for themselves. Perhaps part of the problem with school mathematics is that students are seldom given the opportunity to define their own problematics. Solving well-defined academic mathematics problems in the classroom does not adequately portray the kind of problem solving required in everyday settings, where the situation may be anything but clear-cut.

Ethnomathematics

Although terms like ethnobiology, ethnoscience, ethnomedicine, and ethnohistory are well established in the anthropological literature, the term "ethnomathematics" has a much more recent history (Brodie, 1988). The descriptor "ethnomathematics" was first introduced into the mathematics education literature by D'Ambrosio of Brazil (1985, p. 44), who states that the field of ethnomathematics "lies on the borderline between the history of mathematics and cultural anthropology." He claims that research in this area is important for recognizing that different forms of mathematics may result from different modes of

thought across cultural groups. The "cultural groups" to which he refers are very broadly conceptualized. D'Ambrosio (1985) states:

> In contrast to this [i.e., the mathematics that is taught and learned in schools] we will call *ethnomathematics* the mathematics which is practised among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes, and so on. (p. 45)
>
> "Ethno". . . includes their jargon, codes, symbols, myths, and even specific ways of reasoning and inferring. (D'Ambrosio, 1987, p. 2)

D'Ambrosio (1985, 1987) calls for a wider conceptualization of mathematics that would allow legitimacy for nonacademic practices. He states that the purpose of his research program is to identify a structured body of knowledge within ethnomathematics. The research program is being carried out by collecting examples on the mathematical practices of culturally differentiated groups and linking these practices to patterns of reasoning and modes of thought.

The concept of ethnomathematics presented by D'Ambrosio remains unclear as a number of alternative descriptions are provided to the reader. Ethnomathematics is portrayed simultaneously as the *study* of mathematics within a cultural, historical, and anthropological framework, as well as "the mathematics which is practiced among identifiable cultural groups" (1985, p. 45) (i.e., the mathematics *itself* is also called ethnomathematics). D'Ambrosio gives few practical guidelines as to what specifically should count as "mathematical practices," and there is no discussion of the analytical process that would be required to uncover the patterns of reasoning mentioned above.

The term "ethnomathematics" was coined independently by Ascher and Ascher (1986, p. 125) of the United States. The Aschers define ethnomathematics as "the study of mathematical ideas of nonliterate peoples," by which they mean those people who do not have any system of writing. This use of "nonliterate" is distinctly different from "illiterate," which refers to those individuals who do not have the reading and writing skills possessed by other members of their society. It should be noted, however, that the Aschers agree with a broader use of the term "ethnomathematics," where study would not be restricted to the

mathematical ideas of only nonliterate peoples (Marcia Ascher, personal communication, October, 1988). The work of Ascher and Ascher refutes the widely held views that the mathematical thought of nonliterate peoples is "primitive," is confined to number, lacks analytic thought, and does not lead to abstraction and symbolism. Descriptions of spatial ordering, number systems, and kinship structures from diverse nonliterate cultures point to complex mathematical ideas that are implicit in these activities (Ascher & Ascher, 1986). An account of the logical-numerical system embodied in the quipus developed by the Incas, an ancient, nonliterate South American civilization, is given by Ascher and Ascher (1981).

There have been recent attempts to arrive at a clear definition of "ethnomathematics" (Franco, 1989). There is also a move to come up with a more acceptable name for this area of research, since the prefix "ethno" conjures up the idea of race, which is neither intended nor desirable. This is not the only etymological problem faced by the word "ethnomathematics," since without the correct suffix to denote "the study of," this meaning intended by both D'Ambrosio and the Aschers is sure to remain obscure. Attempts to clarify the definition or alter the term have not met with success so far (maybe "anthromathology" would work?).

For the purposes of this research, I modify Ascher and Ascher's (1986) definition to read "ethnomathematics is the study of the mathematical ideas of any cultural group." I disagree with the use of the descriptor "ethnomathematics" for the mathematical ideas themselves, since this may lead to distinctions being drawn between "ethnomathematics" and "real mathematics." I shall simply use the words mathematics, mathematizing, or mathematical ideas/practices to refer to the content matter. The primary purpose of investigating the different kinds of mathematics generated by different cultural groups is to broaden our conception of mathematics and not to separate and label culture-specific categories of mathematical thinking.

Gerdes (1985, 1986, 1988a, 1988b) expresses great concern about the mathematics that has been generated within the Mozambican culture and that is in danger of being lost.

He asks, "How can this 'pushing aside' and 'wiping out' of spontaneous, natural, informal, indigenous, folk, implicit, non-standard and/or hidden (ethno) mathematics be avoided?" (1988b, p. 3). He addresses this concern by illustrating the many ways in which cultural elements, such as arts and crafts, can be used to investigate mathematical ideas. He asserts that there is "hidden" or "frozen" mathematics in the traditional work done by artisans. For example, the geometrical forms of traditional Mozambican baskets, mats, pots, houses, and fishtraps often demonstrate the optimal solution to problems of construction. These objects reflect not only the accumulated experience and wisdom about the physical materials used but also reveal the result of mathematizing by the original craftsmen who developed the techniques. Gerdes (1986, p. 12) claims that by "unfreezing this frozen mathematics," people can be encouraged that they were capable of developing mathematics in the past and can continue to do so. Gerdes (1988b, p. 6) suggests that pupils could be given opportunities to "reinvent" traditional techniques of, for instance, house building and weaving to "rediscover" and "explore" the frozen mathematics.

As pointed out by Brodie (1988), the mathematical ideas "unfrozen" by Gerdes are basically academic mathematical ideas (e.g., the theorem of Pythagoras underlies the methods of the basket weavers; the properties of a rectangle underlie peasants' house-building practices). While this work is undoubtedly very valuable, we should be careful to acknowledge that there are other forms of mathematics that may not in any way resemble academic mathematics. For instance, Fasheh (1988) observed that his illiterate seamstress mother had a rich and complex relationship to mathematics and that the mathematics she used was integrated into her world in such a way that it remained out of his grasp. Unfortunately, Fasheh does not elaborate much beyond this, nor try to find a way to describe his mother's mathematics that he recognized as being so vastly different from his own academic experience.

The paradoxical situation embedded in the concept of ethnomathematics was pointed out in Chapter 1. The vital question that has not yet been satisfactorily addressed by

researchers interested in ethnomathematics is how to recognize and describe the creation of mathematical ideas that do not fit the traditional mould. As suggested by Gerdes, mathematizing may be "frozen" in traditional handcrafts. The careful study of the everyday actions of people as they engage in traditional practices and day-to-day work activities remains an intriguing possibility through which to find a way of articulating the tacit or "frozen" mathematics that guides such actions.

Harris (1987, p. 27) suggests that a metaphor that implies "hatching or germination of undefined potential" as well as "defrosting" may be more appropriate and more powerful. By saying this, she seeks to highlight the danger that researchers who have had a standard Western mathematics education, with its attitudes and prejudices, may consciously or unconsciously limit what is to be "defrosted" from the artifacts created by persons who have not had the standard Western mathematics education. Ascher and Ascher (1986) explain that the mathematical ideas of nonliterate peoples will have to be found implicit in different areas and activities, since there is no reason to expect that the category of mathematics should exist as a label for all peoples.

In summary, it may be productive to discuss some salient points that emerge on closer examination of the research reviewed in the last two sections.

1. Many of the researchers interested in the mathematical ideas of different cultural groups work in contexts where there is or has been colonization, cultural conflict and suppression, and resistance to oppression (Brodie, 1988). The impetus for these studies has come from a concern about the failure of mathematics education for groups such as ethnic minorities, oppressed majorities, women, girls, and working-class children (Brodie, 1988).
2. The mathematics created by diverse cultural groups and communities, including the communities of fishermen, candy sellers, basket weavers, carpenters, dairy workers, cloth merchants, and tailors, can be studied productively.

3. All people from all cultures perform some forms of the everyday activities of counting, sorting, measuring, comparing, designing, building, and computing.
4. Different forms of mathematics generated by different groups may display varying levels of sophistication (from the point of view of someone schooled in formal mathematics); however, it is important to notice that the mathematics will serve a useful and powerful purpose to its creators.
5. The mathematics of the group is truly owned by its creators, whether or not they are part of the elite group of academic mathematicians. Thus, studying the mathematics of cultural groups could generate cultural reaffirmation as well as individual empowerment through the feeling of ownership.
6. Some attempts have been made to describe the mathematics invented "in practice," but thus far most descriptions have been made within the narrow confines of academic mathematics.

Statement of the Problem

Academic mathematics education has failed for the majority of people. This failure is due in part to the conventional portrayal of mathematics as a prized body of knowledge that is the property of an elite group of people called mathematicians. Research conducted in settings outside of schools shows convincing evidence that people are able to construct mathematical ideas in order to solve problems that they deem to be significant. There is also evidence that school-taught mathematics practices are ignored in favor of strategies that are invented as part of everyday practices.

Rather than dismissing mathematical practices and strategies that develop out of daily activities as lacking in authenticity and rigor, mathematics educators need to study such practices, acknowledging their strengths and seeing their weaknesses as opportunities to negotiate broader understandings in the classroom. Several mathematics educators have called for an acknowledgment that the social, cultural, and political aspects of mathematics and of mathematics education cannot be ignored. An acknowledgment of these factors

would encourage a broader conceptualization of mathematics and may begin a process whereby mathematics could be seen as an active experience, accessible to all people. Working within a broader conceptualization of what counts as legitimate mathematics and incorporating some of the characteristics of the teaching/learning situation of apprenticeship into classroom instruction, it is argued that the usual passive submission to mathematics could be replaced by the active experience of mathematizing.

Research Objectives

1. To document evidence that valid mathematizing is embodied in the everyday woodworking activities of a group of carpenters (who have had little formal schooling) in Cape Town, South Africa. There are two main tasks embedded in this goal:

   a. To understand or recognize the mathematizing in the observed activities.

   b. To describe this mathematizing to the mathematics education community and to anyone else interested in exploring the development of mathematical knowledge.

2. To study the teaching and learning of mathematical ideas that take place according to the apprenticeship model in the setting of the carpentry workshop. There are two main tasks embedded in this goal:

   a. To be accepted as an apprentice by a group of carpenters.

   b. To participate in the practical activities of an apprentice over an extended period of time and to describe the mathematizing that is both explicit and implicit in this learning experience.

Chapter 3

IN THE WORKSHOP

Introduction

As stated, the present study implements the methodology of ethnography. Therefore, this chapter begins by providing a clear description of the context of the research setting. A description of each of the participants is also given. The chapter ends with a brief history of the workshop.

Context of the Research Setting

The carpentry workshop is the place where most of the interactions to be described took place. It was a large, well-lighted room, often bathed in sunlight that streamed through windows situated along two sides of the room. Two big doors on either side of the room allowed a good cross flow of air when the weather was warm. In cold weather, the doors were sometimes closed and a fire was lighted inside in a large metal drum and fed with wood chips. Good-natured complaints about the smoke would then be heard from the neighboring workshop, a small motorcar repair business. The owner of the repair shop was the uncle of one of the apprentices. He repaired the delivery truck for the carpentry workshop as well as the owner's private vehicles. He often forgot to send a bill for the repairs until months after the work was completed. The atmosphere of this small industrial area, which also housed two electrical businesses and an auto body shop, was relaxed and friendly.

There were large stacks of wood piled up along one wall of the workshop: a big consignment of imported Spanish mahogany. A small stock of precious indigenous woods like stinkwood and yellowwood was also kept there. A collection of handtools was visible in a big wooden cabinet against a wall. Machines were dotted around the room—band saws, an overhead table saw, electric planers, thicknessers, a mortising machine, and sanding machines. Next to the long wooden workbenches were two large wooden chests that belonged to two of the qualified carpenters and contained their own personal sets of tools.

Furniture in various stages of completion could be seen everywhere; piles of turned legs and other components were stacked up and ready for assembly. Several beautiful antique pieces—tables, armoires, and chairs—awaited restoration.

There were two adjoining front offices: one for Peter, the owner of the workshop, and one for Emily, his administrative assistant. Both offices were filled with newly made furniture, as well as restored antiques, waiting to be collected by customers. Peter was seldom in his office for long. He could usually be found at work in the workshop, wearing a dustcoat over a pair of casual trousers and a shirt. Emily was kept busy answering the telephone and dealing with accounts, orders, and the weekly payroll.

The working day began at 8:00 a.m. By the morning teabreak at 10:00 a.m., the workshop was an active, noisy place. Against a backdrop of machine clamor, stories were being swapped and lively singing and joking could be heard. Work stopped for 10 minutes each morning, time for tea and a snack. The men gravitated into smaller groups for conversation during the break; the two major racial groups ("coloured"[6] and black) separated, and then within that separation there were further groupings according to status and seniority. At the half-hour lunchbreak and afternoon teabreak, some similar groups were formed; however, many of the men continued working on private jobs (called "spare" or "loose" jobs) at these times. The two daily teabreaks and the lunchbreak were designated by the blowing of a loud whistle, a signal that all the men obeyed without question. In warm weather, some of the men took the opportunity to relax outside in the afternoon sunshine, sitting or lying down under trees that formed part of a scrubby wooded buffer between the industrial area and the trainline. At 5:30, the end of the workday, most of the laborers took

[6]People of mixed African and European descent are classified as "coloured" by the South African government. This label is considered to be derogatory and invalid by the majority of South Africans. For this reason, the British spelling will be maintained throughout this work to emphasize that this is a race classification peculiar to the apartheid system and is not a generally acceptable term.

the train home. Most of the apprentices and the artisans used their own cars or shared rides with one another.

Three languages were spoken in the workshop: English, Afrikaans, and Xhosa. Most of the work-related talk was conducted in English or in a combination of English and Afrikaans. Private conversations were often conducted in Afrikaans and in Xhosa. Most people understood both English and Afrikaans, although a few had difficulty in expressing themselves in these languages. Xhosa was the mother tongue of all the black workmen. Peter, the artisans, and the apprentices (with the exception of Leonard, whose mother tongue was Xhosa) had no knowledge of this language. English is my home language. While not fluent in Afrikaans, I can understand the language and can carry on a simple conversation. I have no knowledge of Xhosa.

Cast of Characters

There were 18 men in the workshop (see Figure 3), including the owner, Peter. Five of the men were artisans, four were apprentices, and the remaining eight were laborers. Only one of the artisans, Clive, had served a formal apprenticeship and had "papers." The other artisans had had varying amounts of informal training and were appointed or promoted to the designation "artisan" by Peter. In the case where a carpenter has no formal qualification, it is up to his employer to give the appropriate financial reward for good quality work. Basically, if someone is paid the wage of an artisan, then he is referred to as an artisan, whether or not he is formally qualified.

The following section describes each person. I have included the race of each worker because this is important for the discussion of social interactions and in the analysis of opportunities for learning. I shall use the terms "black," "white," and "coloured," as they are used in the apartheid system: "black" refers to the indigenous African peoples, whose mother tongue is an African language (e.g., Xhosa); "white" refers to people of European origin, whose mother tongue may be English or Afrikaans; "coloured" refers to people of mixed African and European origin, whose mother tongue may be English or Afrikaans. An

CAST OF CHARACTERS

Owner:	Peter			
Artisans:	Mr. S	Clive	Jack	Jimmy Brian
Apprentices:	Leonard	Patrick	Paul Garth	
Laborers:	Sidney Durham	Ernest Singer	Winston King	Anton Michael

Figure 3. The hierarchy: People in the workshop.

apt quotation is given for each of the six people with whom I had formal interviews. No quotation is given for those people whom I did not interview, and their descriptions are necessarily much briefer. A short history of the workshop follows the description of the workers.

The Owner of the Business

Peter: "There is a skill or an aptitude or an eye in visualizing what you are building which makes you better than others."

Peter was the owner of the business, a white man, fluent in both English and Afrikaans. He was in his late thirties and had three young sons. Woodworking had been a part of Peter's life for many years, beginning in elementary school and continuing through high school, where he took the subject as an elective (not as part of his coursework).

He began his tertiary education studying computer science but later qualified as an engineer. After working for a big civil engineering company in the design office for a while, he began restoring houses. By this time he had begun to collect and restore antiques and to design furniture. His furniture business began as a one-person furniture restoration business, but over time he diversified into making reproductions of classic or antique furniture. Since

reproductions are far more profitable than restoration work, this had become the main line of work at the workshop. However, Peter was particularly fond of antiques and was unwilling to drop the restoration work completely. He had an enviable collection of antiques at home and was well known among antique collectors, who frequently consulted him. The workshop had been in operation at its present site for about two years at the time the present study was conducted.

Peter had no formal apprenticeship in carpentry but had learned informally from skilled artisans over many years. He was a talented designer who enjoyed physically working with wood in the factory on a regular basis. Well liked by the carpenters, he was a good-humored and kind man who was concerned about the progress and well-being of his employees. He added to the relaxed and happy atmosphere in the workshop.

The Artisans

Mr. S: "I found that making furniture was the only thing that would appease my mind, and I became quite good at it. Although I was the youngest of the people he was teaching, when my father died there was not a lot that he could show me. From when I was a child, I stood by his side and watched; he designed hundreds of pieces…that is why it is so easy for me; I can look at an article and design myself a leg that's got to go underneath that article…it just comes…if I think about it, it just comes."

"Geometry—that's the one with the circles and things, isn't it? My daughter learned that at school."

Mr. S, my primary contact in the workshop and my main carpentry teacher, is the key figure in the present study. Mr. S was a Cape Muslim man (classified "coloured" by the South African government), fluent in both English and Afrikaans. He was the chief antique restorer at the workshop. He was the only person in the workshop, apart from Peter, who was addressed as "Mr." by all the other workers, and was supposed to be "in charge" when Peter was absent from the workplace. Mr. S, however, did not enjoy this responsibility. Since he did not have an official title of "foreman," he was not recognized as having any

real authority over the others. He preferred to remain uninvolved in any disputes that arose and saw himself as a loner, with an important measure of independence not possessed by the other workers. He frequently told me that he could make much more money if he restored antiques at home, for his own customers, but that he wanted to help Peter because he was an old friend.

Mr. S was 57 years old and the grandfather of two. All of his forefathers were involved in furniture making or boat building or were silversmiths: his father was a furniture maker; his grandfather, a boat builder; and his father-in-law, a cooper. Mr. S's seven brothers were also involved in the building trade. The roots of the Muslim community at the Cape go back to the late 1600s when skilled tradesmen and artisans either arrived as slaves or were banished to the Cape from Java and Indonesia as political dissidents.

Mr. S began helping his father at home in the family's small furniture business when he was a child about 9 or 10 years old, and at age 13, he earned his first income from woodworking. At this stage, he decided on a career in furniture making and left school. Working at his father's side for many years, through periods of financial ups and downs, Mr. S learned the art of fine craftsmanship. The father-and-son team operated their own private business at home when times were good and customers plentiful. At other times they worked for a prestigious antique auctioneer's gallery in the city. As a young man, Mr. S delighted in examining the "lavishly built" antique furniture displayed in museums and art galleries, and his interest in restoration developed. Together, he and his father developed a good reputation for restoration and reproduction work. In his own words, Mr. S said, "We were sought-after people in this country –we could have made a lot of money in Johannesburg." Of his father, he said, "He was one of the best—and he was a hard taskmaster."

Some years ago Mr. S was asked to teach at the local technical college, but he refused because he was shy and felt that he was uneducated. He regretted the chance he had missed. For this reason, I think that teaching me to work with wood and being involved in

this research project represented a significant opportunity for him. He was confident and knowledgeable and had much to tell about his experiences as a carpenter. It seemed to be a good time in his life to reflect on all that he had achieved. He was approaching retirement age. His youngest daughter was in her final year at school. His other children were successful academically and were all university educated and happily married. There were no major financial worries in his life. Given that he loved reading and books, the idea that he was being written about and that his own knowledge was being valued and his work described was probably very gratifying. He stressed that some of the things he would teach me were secret and that sometimes I would have to work away from the other carpenters so that they could not see what I was doing. His knowledge was presented as being very special and not to be shared lightly with his colleagues.

Clive: "Without pride in your work, you've got nothing. The kind of work that you produce will show whether you are good or not."

Clive was a "coloured" man in his late thirties, fluent in both English and Afrikaans. He was married and had two daughters, one of whom was born during the time of this research. He studied woodworking at high school and then left school at about 16 years of age. After an unsatisfying year of work as a storeman, he decided that his ambition was to work with his hands. He joined a construction company, was endentured as a carpentry apprentice, and qualified as a joiner in just under four years. During these four years, he spent three months of each year at the technical training college where he attended classes, including a mathematics class. For the remaining time, he worked with qualified artisans employed by the construction company in the carpentry section, in the joinery section, and also on building sites.

After qualifying as a joiner, Clive left the construction company and started boat building, which was still his first love. Unfortunately, he developed an allergy to teak wood and had to give up his maritime involvement. He returned to the building trade, where he remained until taking the job as a furniture maker with Peter.

The aspect that he enjoyed most about the workshop was that he created furniture "from scratch," by which he meant that he started with a diagram and rough planks of good quality wood and ended with the finished product. Tasks were generally not fragmented in this workshop as they were in many others that produced cheaper furniture. Clive sometimes worked on the construction of one article of furniture for a week or two with the help of his apprentice. He was extremely energetic and was usually involved with a "spare" job at lunchbreak and afternoon teabreak. During weekends he took on additional woodworking projects to earn extra money. Peter regarded Clive as the best craftsman in the workshop and also the person best able to plan ahead, budget his time appropriately, and solve problems. Peter remarked that Clive always seemed to work very slowly and carefully, with meticulous attention to detail, yet he often finished ahead of schedule.

Clive became another important teacher for me when I began to construct articles. He often walked over to my workplace to correct a small detail of my stance when I was sawing or chiseling, to offer me a more appropriate tool, or to give some verbal advice that made my task easier.

Jack: "Nobody taught me nothing! You've got to steal with your eyes!"

Jack's comment was a very common phrase that I heard from a great number of carpenters and other working-class people from the same community. It referred not only to experiences in the world of work but also reflected the frustration of being denied a fruitful learning experience at school because of the political and socioeconomic constraints of the apartheid system. Jack was a good example of someone who shunned the formal education system at an early age and then actively engaged in self-education, acquiring numerous skills by "stealing with his eyes" from experienced workers in his community. Among other practical activities, he was proficient at welding, braising, mechanical work, panel beating, and spray painting and had made built-in kitchen units and closets. In all these activities, he was self-taught.

Jack was a married man in his late thirties and had a teenage son and another younger son. He was classified as "coloured" and was fluent in English and Afrikaans. As a very young child, from the age of about five years, he began to learn about woodworking from his father and grandfather, who had a small family business making furniture and wooden toys. Jack helped with tasks like putting wheels onto pushcarts and was later shown how to use hand tools and how to operate the electric machines. He maintained an interest in woodwork until he was 15 and helped his brother to continue the family business after the death of his grandfather.

At the age of 16, Jack's schooling was disrupted, and for the next 10 years he worked as a movie projectionist. During this time, he continued to be involved in woodworking projects of various kinds. Eventually, he returned to woodworking to make his living, constructing schoolbenches and laboratory tables. Later he diversified into antique restoration and making other fine furniture. By reading about antique rest n in library books and by observing the mistakes that other people had made on the furniture he restored, Jack taught himself the basics of restoration. He described himself as taking after his father, who was also a "jack-of-all-trades."

He began working for Peter when the factory opened, because he wanted a steady income. Although Jack had customers for the woodworking he did at home and usually worked for his own profit at lunchtime, he preferred the security of a weekly wage to the prospect of relying entirely on his private work for his livelihood. Like Clive, Jack enjoyed the kind of woodworking he did at Peter's workshop, commenting, "When you work in any other factory, which I have seen and I know, if a man screws in screws today, he does that for the rest of his life; if he joins up drawers today, then he makes drawers for the rest of his life. Here you work from the floor right up to the finish of that particular item, and that is what I call a cabinetmaker…a craftsman."

Jack was the third artisan who became a significant teacher for me as time went by. Jack, Clive, and Mr. S were the most experienced artisans in the workshop and were the

people with whom I spent the most time. I conducted several formal interviews with each of them.

Jimmy. He was about 25 years old, unmarried, and was classified as "coloured." Jimmy spoke Afrikaans and had difficulty expressing himself in English. He had not served a formal apprenticeship for carpentry, and the exact circumstances of his informal training seemed to be a bit of a mystery. He made frequent mistakes that Mr. S often quietly helped to put right. The other artisans did not value his workmanship highly. In fact, they cited his work as an example of the work of a poor carpenter. One of their criticisms was that he did not have his own set of tools, which was very unusual for an artisan. According to Peter, Jimmy "has a very nice personality and works hard, but doesn't grasp things too easily."

Peter told an amusing story about how Jimmy arrived for a job interview, very keen to have the job, and toting a toolbag. Peter decided to check on what tools he had, as one of his criteria for hiring, and asked Jimmy if he had a rebate plane. A rebate plane is an expensive item, and if somebody owns one it usually means that he has a very good set of tools. It turned out that Jimmy did have one, and it had a clean, sharp blade, which implied that he looked after his tools. Peter was so impressed that he hired Jimmy immediately, then found out later that he did not have any other tool besides the rebate plane and still does not own any other tools to this day!

Brian: "Gee, that took a long time for me to teach it to you, but that is because I had to teach it to myself first!"

Brian was about 21 years old, white, English-speaking, and newly married. Peter told an interesting tale about his meeting with Brian and the latter's inauspicious start to a successful career in woodwork. Brian was about 18 years old and had no woodworking skills when Peter first hired him to help paint his home. When Peter decided to start the workshop and began looking for staff, he was told that Brian was interested in joining. At first, Peter was suspicious, since carpentry is traditionally the domain of "coloured" men in

South Africa, and Brian, as a white, had much better career opportunities to choose from. However, Peter finally decided to give Brian a chance.

At first, he was disappointed; Brian had overstated his high school qualifications somewhat, as became evident when Peter tried to enroll him as an apprentice. He had only the equivalent of grade seven, which was below the minimum required for embarking on a formal apprenticeship. Furthermore, he displayed no interest in anything at the workshop, often arriving late to work in the morning.

At that time, Peter used to turn all the legs needed for tables and chairs himself. All this work was done on a lathe, using hand-held tools. This was very time-consuming and needed special expertise. According to Peter, he realized one day that he could not keep up with all the turning, and so he turned to Brian, and "I said to him, why don't you try it? He immediately took to it…he has the eye for it…he has a very good eye. He turned out to be tremendously successful on the lathe!" On the strength of Brian's high output and profitability, Peter decided to pay him an artisan's wage, and thus he became an artisan with no formal training.

The other carpenters told another story about Brian and his skills on the lathe, a story that revealed some of the work ethic of the workshop. Brian had not yet been made an artisan, although his rapidly developing prowess at turning legs was impressive. He was able to work very fast and accurately. Peter noticed this and jokingly told Brian one morning that if he could complete a certain number of legs by the end of the day, he would be paid an extra sum of money. Brian countered, saying that if he finished the legs by lunchtime, he should get the money as well as the rest of the day off! Peter agreed, and the wager was set. Some of the other carpenters tried to dissuade Brian, saying that it was a bad idea to show the boss how much he could really do if he tried hard; if he succeeded, then Peter would always expect such a high output! Brian, however, could not be dissuaded, and worked at top speed, finishing even more legs by lunchtime than Peter had specified. He was given his extra money and the rest of the day off, and shortly thereafter, he was made

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an artisan. But as the men predicted, his level of output has had to remain high because of the expectations he set. He seemed to enjoy the work, despite the pressure to produce, and singlehandedly turned every component that needed to be worked on the lathe.

The Apprentices

Leonard: "What I make is my own. I can do it according to my plan in my head."

Leonard was Xhosa-speaking, an unmarried black man in his early twenties. He had a matriculation certificate (equivalent to grade 12) with mathematics, science, and two languages, which meant that he had by far the highest academic qualifications of all the employees in the workshop. Leonard had a great deal of ambition and confidence. Peter thought highly of his abilities and was especially encouraging towards him.

He began as a laborer at the workshop, sweeping, polishing, and scraping. After about six months, he informed Peter that he wanted to become an apprentice. He began working on his own woodwork projects during breaktimes, asking Peter for help when necessary. He claimed that his "standard was higher" (than that of the other laborers) and thus he deserved to be trained as a carpenter. Peter agreed, but he encountered difficulties when he tried to enroll Leonard at the local technical college. The apartheid system keeps race groups strictly segregated in the education system—different institutions are set up for each race group. As previously mentioned, carpentry has traditionally been the turf of "coloured" men, and there are few black carpenters. The technical college would accept only "coloured" students, and no facility existed in the city for black students to become carpentry apprentices. Finally, the education authorities relented and accepted Leonard as an apprentice. He left the workshop to attend the technical college full-time for a three-month period about five weeks after I started work at the workshop. Before he left, I spoke to him about his early life and his ambitions.

When Leonard was a young boy, he used to make toys and miniature cars out of wire and pieces of Coke cans that he cut up to form small metal sheets. He and his friends once made a full-sized car from scrap metal and wood. It had wheels taken from

wheelbarrows and a steering wheel that operated a steering device. The group of friends enjoyed pushing their "car" around and traveling downhill whenever possible!

Leonard described the articles he had made during the breaks at the workshop, saying that he "works from a plan which is in his head." His completed projects were two jewelry boxes and a cassette tape holder, and his most recent project was a wooden bread box with a special sliding door. The design for the bread box was copied from something similar that he saw in a store. Using his outstretched hand, he was able to measure the dimensions of the box in the store, and an examination of the way in which the sliding door works enabled him to make a replica of the box in the workshop, using scrap wood.

Leonard obviously enjoyed woodworking very much and particularly commented on the fact that he owned the things that he made. He told me that he worked as a laborer in a flour mill for a few years and came to the realization that he would never own a mill himself. This realization prompted his decision to learn a trade that would allow him to make something of his own, with his own ideas.

Patrick. Patrick was about 20 years old and unmarried. He spoke English and Afrikaans fluently and was classified as "coloured." It was said in the workshop that he was Peter's favorite and so was allowed to get up to all sorts of mischief. Peter regarded Patrick as exceptionally talented and possessing an excellent grasp of spatial concepts, but he acknowledged, however, that Patrick was lazy and full of youthful rebellion. Peter anticipated that Patrick would settle down as he got older and that he would develop a more mature attitude towards his work.

Like Brian, Patrick had overstated his high school achievements when he applied for the job at the workshop and was not allowed to enroll at the technical college as an apprentice. Eventually, he was accepted in the category of "learner" by the Industrial Council, which would allow him to study at the technical college and to obtain a qualification in carpentry. This career path had a ceiling, however, and he would not be able to qualify as a master carpenter.

Paul. Paul was a year or two older than Patrick; they were close friends and shared an apartment. Paul was classified as "coloured." He arrived back from a three-month study period at the technical college soon after I arrived at the workshop. According to Clive, Paul was not really dedicated to his work, although he seemed to do adequately. He sometimes spoke to me of his ambition to teach carpentry at the technical college after qualifying, because he did not relish the idea of being in a workshop for the rest of his life. His uncle owned the mechanic's workshop next door. Paul and Patrick made a formidable team of pranksters in the workshop.

Garth. Garth was English-speaking, classified as "coloured," unmarried, and in his early twenties. He was quiet and reserved and went about his work seriously without joining in the general horseplay. Clive, the artisan with whom Garth worked most often, regarded him as the most promising of the apprentices. The personal project on which he spent most of his lunchbreaks was a beautiful chess board, inlaid with pieces of light and dark wood.

The Laborers

Sidney. Sidney was a very dignified Xhosa-speaking man approaching his seventies, who had a wife and children in the Ciskei homeland. He had known Peter for 20 years and had worked for him for 18 of those years, helping to restore furniture. Peter went to Johannesburg for a period of 2 years, during which time Sidney went to work for another restorer. When Peter returned to Cape Town, Sidney rejoined his business. Sidney lived in a small apartment that was attached to Peter's home.

He actually did no woodworking at all, although he was extremely knowledgeable about wood and was an expert at identifying different types of wood. He was the workshop's authority on aging the newly replaced parts of percentage restorations (i.e., making the newly replaced portion look as old as the original). He also threaded "riempies" (thin strips of tanned hide) to make chair seats.

Sidney was well respected by everyone as an elder and was seen as being "in charge" of the laborers. He kept time during the day, blowing the whistle at the beginning

and end of tea and lunch breaks, a signal that everybody obeyed. Sidney was the uncle of Leonard, the apprentice previously described, and of Ernest (Leonard's brother), who is described next. Peter relied upon Sidney to recommend workers when a new laborer was needed. All of the black laborers at the workshop were introduced to Peter by Sidney.

Ernest. Ernest was Xhosa-speaking from the Ciskei, unmarried and in his early twenties. During the time I was at the workshop, he made two jewelry boxes using a light wood with inlays of a darker wood in the shapes of playing card suits. His work was quite rough, and attractive in a rustic way. The artisans criticized his workmanship rather cruelly but made no effort to assist him. He was much more outgoing and friendly than Leonard but seemed to lack his brother's confidence and resolve. He had not been singled out by Peter as being especially capable and was still a laborer.

Winston and *Anton* were Xhosa-speaking men from the Ciskei homeland and were brothers. Anton was about 20 years old, very quiet, and worked as a laborer. Winston was a little older and had been taught to do French polishing by Mr. S. He usually walked to the store every morning, armed with a long shopping list of everyone's daily food requirement, and returned in time for tea to distribute the food and correct change.

Durham was about 50 years old and was Xhosa-speaking. He was the truck driver for the workshop and a good friend of Sidney. He sometimes helped Sidney to thread riempies, to age wood, and to polish.

Singer was about 25 years old and was unmarried. He was a Xhosa-speaking man from the Ciskei. Mr. S taught him how to French polish, and this was his main task. He was frequently exhorted by the others to "get a wife," because he washed his clothes at work and hung them out to dry on the fence outside! Singer was a keen amateur boxer and had a great sense of humor. He sometimes did the morning shopping instead of Winston.

King was about 30 years old, Xhosa-speaking and unmarried. He had previously worked for Mr. S, and they joined Peter's workshop together, with King as Mr. S's assistant. He was a laborer and was quiet and reserved. During breaks, he worked on his

personal woodworking projects and made a coffee table from off-cuts and later made a small bedside cabinet with a mirror.

Michael was a laborer in his mid-thirties and was Xhosa-speaking. He was shy and was one of the main butts of Mr. S's teasing and horseplay. He worked extremely hard in the workshop and was regarded by Peter as the most diligent worker.

History of the Workshop

The business began at Peter's home two years and six months ago. For six months, Peter, Sidney, and Mr. S did restoration work only; then Peter began to look around for premises and to purchase machines. Two years ago, "The Joinery" (not the actual name of the business) was established at its present premises with a work force consisting of the original three, plus Paul and Patrick, who had helped paint Peter's house, and their friend Brian. The last three were all about 18 years old and had no woodworking skills. Patrick and Brian had both inflated their schooling qualifications as previously explained and could not be enrolled as apprentices as planned.

Brian's special skill on the lathe earned him rapid promotion to artisan status, and Patrick became a learner carpenter. Peter advertised in the newspaper and hired Clive, Jack, and Jimmy. Clive had good credentials; he had worked for one of the leading builders in the country who had decided to close down his business when he retired. Jack was hired on 10 days probation, but Peter asked him to stay after 2 days because he was impressed with his work. As previously related, Jimmy got the job on pure luck! Almost all of the laborers were members of Sidney's extended family or were friends of his.

The men worked a five-day week and a nine-hour day, from 8:00 in the morning until 5:30 in the afternoon, with half an hour off for lunch. The atmosphere was happy, and there was a lot of lighthearted banter and good humor. Apartheid had put its seemingly inescapable and indelible stamp upon the social interactions, and some subtle and other not-so-subtle forms of segregation and discrimination were practiced by some of the workers. The social issues will be discussed in more detail in Chapter 6.

Chapter 4

METHODOLOGY

Introduction

The present study was conducted over a period of six months in Cape Town, South Africa, where I worked as an apprentice with a group of carpenters. Although I learned both carpentry and new ways to mathematize from many of the carpenters, as mentioned earlier, I had one particular teacher, Mr. S, with whom I worked on a daily basis. He taught me the basics of furniture restoration and woodworking, as he would have done any apprentice.

In Chapter 1, I outlined my reasons for selecting an ethnographic approach and for the importance of my position as apprentice. In this chapter, I comment generally on the use of ethnographic approaches for educational research. The difficulties of my entrée into the field are sketched out, and my experiences in the workshop are elaborated as I was initiated and gradually accepted as a useful worker. A description is provided of the methods used to collect data. Throughout the chapter, and particularly in the second-to-last section, I point out some of the frustrations that a researcher encounters when she is intent on doing naturalistic inquiry in a noisy, busy workplace. I conclude the chapter by describing my exit from the field.

Ethnography and Educational Research

Ethnographic research has its roots in cultural anthropology and sociology. Researchers have found ethnography to be a difficult concept to describe precisely. However, the theme that stands out clearly in all discussions of ethnography is that the setting of interest for the research concerns the interaction of humans together and the meanings that they create together.

Interpretations of the role that ethnography plays in the context of education can be gleaned from the words of prominent ethnographers. Wolcott (1987) maintains that the purpose of ethnographic research is to describe and interpret cultural behavior. Goetz and LeCompte (1984, p. 51) talk of ethnography as being the "holistic depiction of uncontrived

group interaction over a period of time, faithfully representing participant views and meanings." Spindler and Spindler (1987) claim that ethnographers attempt to make a coherent recording of how people in a community behave and how they explain their own behavior. They emphasize the difference between ethnography and inference, stating that the ultimate purpose of ethnography is to provide reliable source material for analysis, which is governed by systematic models, paradigms, and theory and out of which inference will flow. Inference and speculation are therefore grounded in observation and theory.

Ethnography is usually concerned with the development of a theory and not with the testing of existing hypotheses. Glaser and Strauss (1967) introduced the idea of "grounded theory," which refers to the discovery of theory from data that has been systematically obtained from social research. An essential part of the research task is to discover what is significant and what is important to observe. One is thus continuously involved in a process of inquiry. Fieldwork and interpretation are done concurrently, rather than being sequential steps in the research process. Glaser and Strauss (1967) strongly recommend that for research aimed at discovering theory, the three procedures of theory building, coding, and data analysis should go on simultaneously.

During the past decade, qualitative research in education has become increasingly acceptable as an alternative to quantitative and positivist models. Educators often turn to ethnographic approaches to inform their research questions. The literature contains many criticisms of educators who presume to call any attempt at descriptive work "ethnography" without paying attention to the philosophic and theoretic roots of the approach. Rist (1980, p. 9) describes "blitzkrieg ethnography" as a " 'hit and run' foray into the field," while Wolcott (1987, p. 40) writes disparagingly of "would-be ethnographers" who provide wonderful descriptions but who fail to analyze what they have observed, and "gonzo-ethnographers" (1987, p. 45) who do not pay enough attention to the concept of culture.

Eisenhart (1988) gives an excellent discussion about the ethnographic research tradition and mathematics education research where she points out that the interpretivist

philosophy underlying ethnography is very different from the logical positivist ideas that usually guide traditional educational research. She identifies two broad groups of researchers doing descriptive studies of mathematics education: traditional mathematics educators and educational anthropologists. Whereas traditional mathematics educators usually have goals that are "descriptive and prescriptive" and tend to ask research questions in the vein of "How can mathematics teaching and learning be improved?" educational anthropologists interested in mathematics have goals that are "descriptive and theoretical" and ask "Why is mathematics teaching and learning occurring in this way in this setting?" (Eisenhart, 1988, p. 100). It should be made clear that although the outcome of research by educational anthropologists with an interpretivist philosophy may not directly prescribe a remedy, an understanding of the problems can lead to appropriate actions being taken.

Educational researchers holding the interpretivist view try to capture and share the understanding that participants in a social setting have of their knowledge, of what they are teaching and learning. An important idea in interpretivism is that all human activity is fundamentally a social experience where people try to make sense of their worlds (Eisenhart, 1988). This idea concurs with the constructivist view that people construct their own mathematical knowledge through their actions and reflection on problematics that have particular significance to them (Confrey, 1985). The meaning of this knowledge is negotiated and agreed upon among participants in the social setting of a community.

Obtaining Entree

The first and most difficult part of conducting this research was to find and gain access to a suitable research setting. Locating a carpenter who was prepared to accept me as a learner was no easy task, and it was a full month before my searching was rewarded with the sweet smell of success—sawdust! I lived with a family in Athlone, Cape Town, in a neighborhood that is reserved for "coloured" residents in terms of the Group Areas act, one of the foundations of the apartheid system. It is an area where many artisans reside. For various reasons, there is less of a stratification along socioeconomic lines in this

neighborhood than would generally be found in the U.S. Blue-collar workers may have children with professional or academic positions who settle in the same neighborhood.

Through members of the family with whom I lived, I began to build a social network of people who might be able to introduce me to carpenters. For several reasons, I particularly wanted to work with someone who had been trained in an informal apprenticeship with family members and who had his own small business. I thought that someone with a small family business might be more flexible and thus more likely to accept me as a learner. I was not interested in working in a highly mechanized workshop, as some of the larger ones tend to be. I did not want to be taught by somebody who had received formal training at a technical training center, since academic mathematics is taken as part of the coursework. Since the Cape Muslim people are renowned for their centuries-long tradition of excellent craftmanship in fine furniture making, I especially tried to meet people from this cultural background.

I met some very kind and helpful members of the community who tried their best to assist me in locating a suitable research setting. There were many disappointments and dead ends, and on several occasions, I was tempted to give up my search for the elusive carpenters. It was during this time that I learned the true meaning of the words "frustration," "perseverance," and "tenacity" ! I visited several furniture factories and restoration workshops, but nobody was prepared to accept me as a learner. In some cases, there was resistance to talking to me because of my gender. I was told that it would be against factory regulations for me to work in a workshop. I also began to realize how noisy and rushed the atmosphere is in a workshop where most of the work is being done by machines. Sometimes it is absolutely deafening. Conversation is impossible—in fact, the workers wear earmuffs to protect their eardrums! Noise level was not mentioned in any of the articles I had read about doing research outside of an academic environment. It is definitely something to be taken into account when planning research of this nature.

Many of the carpenters who had learned their trade in the traditional way from their fathers and grandfathers and who had operated small family businesses from their homes had died during the preceding five years. I was often told about a member of someone's family who would have been the perfect teacher for me but who had unfortunately passed away. I began to think that I had started this study five years too late! Small family businesses seemed to be a thing of the past. One "coloured" furniture factory owner described the difficulties faced by black and "coloured" entrepreneurs when trying to acquire finance to start a business. This particular factory owner had struggled for many years against daunting discriminatory practices to set up his business and to secure premises. The result is that most of the businesses making top quality furniture are owned by whites, who employ mostly "coloured" master craftsmen and black laborers. I decided that I would have to consider workshops of this last type as possible research sites too.

After two weeks of following up every lead, sometimes at strange times of the night, and of making countless telephone calls, I heard of a master craftsman, a Cape Muslim man, who was working at a white-owned restoration and reproduction workshop. I called the owner of the workshop and explained a little about my research and asked if I might speak to the master restorer. The owner, Peter, was open to the suggestion, and I arranged to visit the workshop the next morning. I spent a very interesting hour with Mr. S, who had exactly the background that I was seeking and who obviously enjoyed showing me round the workshop and talking about his life. He had worked with Peter for two years but had been acquainted with him for longer, and he obviously had senior status in the workshop. It so happened that he lived near some of the people who were part of my social network. We arranged that I would visit him at his home in 10 days time so that he could show me some of the furniture he had made by hand. He had a small workshop at his home. I was very excited to hear about that, because I thought there might be a possibility of my being able to work at his home. I was afraid to place too much hope on this, however, based on my previous disappointing experiences. During the next 10 days, I continued to make contact

with a wide variety of people, meeting several interesting craftsmen who were prepared to talk to me but none who were interested in teaching me.

My visit to Mr. S's home exceeded all my expectations. I was introduced to his wife, children, and grandchildren. We went directly into the living room and spent two uninterrupted hours in discussion. Mr. S was very talkative and rapidly jumped from topic to topic as if he wanted to tell me everything he knew all at once. His house was filled with magnificent furniture that he had made by hand or restored. He told me that he was "not educated" but had read a lot of books and articles about furniture and loved old English furniture. He had "stolen with his eye from books" and made modifications to suit his own taste. He had done some beautiful hand carving on large bedroom cupboards and took great delight in showing me how different his design was from any other. He enjoyed making something unique. He drew my attention to tiny details that I had not noticed.

We had a delicious lunch with the whole family, eating traditional Cape Muslim dishes that had been prepared by his wife. After lunch, we again retired to the living room to continue our discussion about Islam, politics, history, education, and Mr. S's work. He showed me his own research: folders containing cuttings from a great variety of sources, with articles about furniture design and restoration, wood carving, and reproductions. There were several diagrams he had drawn, showing designs for decorations. He clearly had a great interest in and love of his work and enjoyed reading. We looked at his small workshop, which was overflowing with wood and old furniture. Not much room for a student, I noted with disappointment!

He showed me some carved jewelry boxes that he had made for his daughters. Tentatively, I asked if he thought I would be able to make something like that. He was most enthusiastic and started to explain how he had made them. I told him that I would like to learn to do the practical work myself and that I would like him to teach me. To my amazement, he said that I should come and work with him at Peter's workshop. He proposed to start me off learning like an apprentice, sanding and polishing, and then go on to teach me

how to make a jewelry box and many other things. He advised me to phone Peter in the morning to make sure, but he seemed absolutely certain that Peter would agree. Finally I left, delighted with the turn of events and exhausted after six intense hours of conversation.

The next day, I decided to allow some of the morning to go by before phoning Peter, hoping that Mr. S would be able to broach the topic of my being taken on as his apprentice. When I telephoned Peter and asked if I could come and talk to him, he seemed most amenable. Mr S was at the entrance when I arrived, working on a big table. He whispered to me that he had "laid the foundations for me" with Peter, which indeed he had. I had to wait a while for Peter to finish dealing with a customer, and during this time, Mr. S took me round the workshop and introduced me to several people, saying, "This is Wendy—she'll be with us for a while." Needless to say, I was most encouraged by his confidence in his "foundations."

I briefly explained the gist of my research questions to Peter, emphasizing the necessity of my role as a learner and participant in workshop activities. He agreed to allow me to work with Mr. S. He seemed intrigued and slightly amused by the idea but offered to help in whatever way he could. We arranged that I would work at the workshop on a daily basis for the next five months, for approximately six hours a day, starting the next day. I decided to reserve the latter part of each day for writing fieldnotes at home. Peter made it clear that he intended to leave the organization of my work up to Mr. S and that I could stay as long as my presence did not hinder production in any way. We agreed that he would ask me to leave if he felt that the carpenters' work was being compromised by my research activities.

Initiation and Acceptance

Mr. S asked me what I wanted to start with, and I suggested that he give me whatever he would usually give to a new apprentice. He looked at my hands and remarked "Polish is going to spoil your hands." I assured him that I did not mind. When I arrived at work the next day, I found my first job neatly laid out on a small workbench. My task was

to sand a polished walnut lampstand down to bare wood and then to French polish it again to high gloss.

At first my workbench was on the outskirts of the workshop. During the first week it began to inch forward into the main work area. I was reserved and a bit anxious about approaching the men unless I had a specific question in mind. They approached me at first—first Jack, then Patrick, later Clive and the others. On my second day at work, Patrick came over to my worktable and stood and watched me for several minutes as I labored over the tall walnut lampstand. The lampstand had interesting curved details and was difficult to sand evenly. It was a hot day. My back hurt, my face was flushed and dusty, and my hair kept falling into my eyes. Patrick continued to watch me closely. He said tentatively, "Wendy, wouldn' t you rather learn to sew?" "No," I replied firmly, "I want to learn to do woodwork." Patrick digested this for a while. Finally he observed, "I suppose you must be one of those tomboy-girls."

For the first few weeks, Mr. S constantly inquired whether or not I was in a draft, agonized over the fate of my hands, which were becoming hardened and coated with dark brown shellac, and handed me small cloths to wipe the dust from my face. The other men also paid attention to my hands and warned, "Your hands will be harder than your future husband's hands!" Slowly their interest waned as they became accustomed to my presence.

One of my initial problems was to find a way to have tea and lunch with the carpenters. There was a half-hour break at lunchtime and two 10-minute teabreaks, one at 10:00 in the morning and the other in the mid-afternoon. I worked from 8:30 in the morning until 3:00 in the afternoon, so I had two opportunities to converse with the workers "on their own time." At all other times they conveyed the feeling to me that we were "stealing the boss's time." Sidney kept a meticulous eye on the clock and blew a whistle to signify the beginning and the end of each breaktime. Everybody obeyed this signal.

Joining the workmen at breaktimes was awkward for several reasons. One unexpected obstacle was the friendliness of the secretary, who was a young woman eager to

have company during the breaktimes. She sat alone in an office attached to the workshop. She made me tea and brought me good things to eat from her home. The more significant related problem was that she was not the only one who expected me to keep her company. All the carpenters also expected me to have tea with her, since she was the only other female member of the staff. Mr. S commented, "You can have a break from us men and have a woman to talk to."

Peter expressed his opinion that my presence in the workshop during breaktime might represent an intrusion of the men's privacy. He suggested that the men might feel inhibited if I sat with them while they were eating and talking and might be unable to relax adequately before their next period of work. The clear segregation practiced by the men was easy to observe, and I did not want to join any one particular group. The black laborers sat and ate together as a group and had little interaction with the others. The "coloured" artisans formed another group, and the apprentices another group. Interaction occurred between the artisans and the apprentices. There were loners from all three of these groups who sat apart from the others, one of these being Mr. S. The groups were formed mainly at morning teatime, which was when most people ate. At lunchtime, many of the men were engaged in personal woodworking projects.

After about 10 days I realized that the way to legitimize my presence in the workshop at tea breaks and lunchtimes and to avoid all the problems mentioned was to have a "loose job" or "spare job" like everyone else. This meant having a job that was for oneself or for personal profit. My engagement in a woodworking task would provide a clearly acceptable reason for spending the breaks in the workshop and would furthermore put me in a position to cultivate conversations with whomever I pleased. Under the direction of Mr. S, I began to work on a carved jewelry box made from yellowwood with an inlaid star design on the lid made from stinkwood and bone. Both woods are precious indigenous woods. Peter allowed everyone in the workshop to use small off-cuts for private projects. Mr. S designed the box, making sure that I understood everything he was doing and including me

whenever possible. I enlisted the assistance of several workers besides Mr. S when I began the construction.

As time went by, it became easier to join in the conversations at breaktimes and after a couple of months I became Mr. S's constant companion during morning teatime and lunchtime. The secretary left after my first month there. Although I was sorry to see her go, this event made my presence in the workshop at all breaktimes seem even more sensible to the men, since I no longer had the choice of a female companion. When the new woman secretary arrived a week later, nobody suggested that I should keep her company during breaks.

After six weeks of scraping, sanding, and French polishing, I moved into a different stage of my apprenticeship. Mr. S told me that I had done enough of that sort of work, and he began to treat me more like his personal assistant. My physical position in the workshop changed too, from being on the outskirts and creeping in, to being, in Mr. S's words "in the thick of things!" I worked closely with him, cutting wood, fetching things, gluing, clamping, drilling, and chiseling. He stopped worrying so much about the state of my hands. I started work on replacing part of a stinkwood and yellowwood inlaid table, where I had to replace a missing drawer and do the inlay work. I later made a second drawer with dovetail joints. At this stage Mr. S began to refer to me as his "newest apprentice" when talking to the other men and to customers at the workshop. Some of carpenters noticed and commented on the change. Paul grumbled that I was getting far better care than he had as Mr. S's apprentice! When I spent the day away, several of the men would teasingly demand an explanation for my absence. Clive commented, "We work five days a week here, Wendy, not only four!" I learned to tell several of the men, and not only Mr. S, ahead of time if I intended to change my accepted work schedule for any reason. Mr. S commented on how much they all missed me when I was absent.

During my third month at the workshop, I completed two jobs that seemed to earn me a lot of recognition. A mahogany sideboard had been designed, built, and polished in the

workshop. Several workers had been involved in the construction and finishing of the sideboard. It was a special piece, custom-made for someone. It had four drawers for storing silver cutlery. My task was to cover the cutlery holders that fit into the drawers and the interior of the drawers in green baize. The task was definitely something categorized in the workshop as a "woman's job" and would have been done by Peter's wife at home if I had not been available. It required a lot of fine work with a razor blade, tweezers, and glue. All the workers came to look, to check on the neatness, to admire, and to give advice. I am not sure why this drew so much approval from the group. Maybe because it was a special item of furniture and the green baize lining put a fine finishing touch to the whole piece. Since a number of people had been involved in the project, there was pride in the work of the whole team. This generated a great feeling of camaraderie.

The other project that drew many favorable comments from the group was the construction of a stereo stand that I made for a friend. This represented a "spare job" for which I was being paid. Although it was a "spare job," I got permission from Peter to work on it full-time for a few days. Mr. S and Clive directed my work and gave assistance while Paul and Patrick, among others, watched my progress and made comments. Clive and I selected the rough planks and then working from a diagram drawn by my friend, cut the planks to size, thicknessed and sanded them, made the joints, glued the components together, and squared and clamped the construction. I sprayed the stand with sanding sealer and then polished it. This was the first time I had played a major role in a visible project that "started from the floor up" (i.e., worked from rough wood, constructed the article according to a specific design, and spent considerable time on the finishing). Although my first personal project was the construction of the jewelry box, much of this work was invisible to the men, since Mr. S insisted on keeping the box hidden, for fear that the other carpenters would copy his design. I worked on the box at home, or kept the components separate when I worked on it at lunchtime in the workshop. Nobody, not even Peter, saw the finished jewelry box.

Establishing myself as a bona fide worker was a slow process, but it was accelerated after the first six weeks when my workplace changed from the outskirts to the hub of the workshop. The nature of my work also changed after the first six weeks, and I was expected to cut wood with the electric saws (under Mr. S's watchful eye), use the electric drills and sanding machines, use a handsaw and chisel to make dovetail joints, and become involved in projects requiring inlay work and other construction. Because of these changes, I had greater access to the men during the working day, and it seemed natural for me to spend breaktimes with them and to be included in their conversations. Peter paid me for my work from the beginning, and this also helped to legitimize my labor. Mr. S, Jack, and Clive always asked me how much I had been paid for each job that went out to a customer. Judging by the comments of all the men, I was accepted as a participant during the third month. As the weather grew colder, I was included in the group that sat around the fire drum at breaktimes. The workshop was high ceilinged and drafty, and we enjoyed toasting our sandwiches and warming our hands at the fireside.

Although I had explained the general goals of my research to the workers at the outset and had talked about my degree program to them from time to time, it was interesting to hear them construct their own explanation as to why I was in the workshop. The time that apprentices spend in the workshop, between periods of study at the technical college, is referred to as "doing their practical." When a visitor expressed surprise at seeing me (a white woman) working in the workshop, or asked someone what I was doing there, s/he would be told, "Wendy is studying in America, but she has come home to do her practical. When she is finished here, she'll go back to college in America again." In this way, my research was given meaning that made sense in the context of the workshop.

After working on a daily basis for just over three months, I left the workshop for three weeks for a period of writing, thinking, consolidation, and planning. On my return, I received an enthusiastic welcome and was put to work immediately. There were many absentees because of illness, and I was needed as a worker. I went along with this to a

certain extent but felt that my research was being compromised. Also, I was conscious that I would be leaving within two months and felt that I wanted to phase myself out slowly to avoid an abrupt departure. However, there was a lot that I could do to help, and for several days I played the part of a worker and performed solitary tasks that did not give me access to any of the other carpenters.

When Mr. S went away for a few days, I asked Jack if I could help him construct tables, which was his specialty. I also began to help Clive with the construction of chairs, his chief task at that time. In this way, I was able to keep two conversations alive as I moved as a helper from Jack to Clive. My projects led to numerous interesting interactions with many of the men, some of which have been described in Chapter 5.

I have tried to describe my initiation and acceptance at the workshop in the context of the culture of the workshop. The workers, although all born and raised in South Africa, represented several different language and race groups and at least two different religious groups. Despite these differences in backgrounds, this working community had spun definite "webs of significance" (Geertz, 1973, p. 5), with its own jargon, habits, traditions, beliefs, expectations, patterns of behavior, and systems of symbols. From my point of view as a South African, conducting research in a country like South Africa, where the political and economic dominance of one group over all others has been entrenched for so long, the concepts of power and conflict are essential in a discussion of culture. With reference to Giroux (1981), the notion of what constitutes culture for this study encompassed also the experiences of these people in their struggle for political power, their opposition to economic inequity, and their ability to shape their own lives in the face of severe economic and political restrictions. It would not be possible to make sense of the teaching and learning situations in the workshop without considering the prevailing social, political, and economic constraints and the limits that these constraints impose on learning opportunities.

Focus on Problematics

Throughout my time at the workshop, I focused on mathematical problems that arose spontaneously during carpentry activities. Working within the theoretical framework of constructivism, I claim that the problems that arose in the carpentry workshop became problematics or "roadblocks" for the carpenters through my questioning. The "Episodes" described in Chapter 5 are examples of the engagement of the carpenters in the constructivist cycle of action and reflection. The acknowledgment of the problematic by the carpenter (manifested by a willingness to continue our conversations and to search for a solution), served as a twofold "call to action." From a methodological point of view, it was a call to me as a researcher to pay special attention to what was happening, a signal that I might witness mathematizing. To the carpenter, acknowledging his problematic was a call to remove the roadbock, to become involved in a cycle of action and reflection in an attempt to reach resolution and restoration of equilibrium. As a novice, I was expected to ask questions and so could seize opportunities to begin discussions that I anticipated as fertile ground for exploring the mathematical ideas of the carpenters.

Once I had located this particular group of carpenters who were prepared to teach me and had shown that I was prepared to work hard, my gender no longer appeared to be an obvious obstacle. As a woman, I was not expected to know anything about a man's work. I could thus ask questions over and over again, encouraging the carpenters to reflect on their explanations, without being scorned for not understanding the first time. It did not seem strange to them that I could not, for example, calculate the volume of a piece of wood, despite the fact that they knew that I was a mathematics teacher. Such a calculation was seen as a technical matter, meant to be solved only by someone (definitely a man) who had specialist knowledge. I was thus free to pursue this question at great length and able to encourage the carpenters to talk about their concepts of volume and the measurement thereof.

Of course it is not possible to predict what would have happened if anyone else had asked these questions or what the discussions would have sounded like if I had not been there at all. My presence as a white woman worker in a carpentry workshop where all the other workers were men, and mostly black men, is unusual for South Africa, where separation and discrimination by race and gender abound. Did the men respond positively to my questioning because I am white and thus in a category to be treated with deference according to apartheid indoctrination? Or did they enjoy sharing their knowledge with an "ignorant" woman? Or perhaps our interactions resulted out of the friendship that grew as a result of mutual respect, admiration, and many shared jokes? I do not know the answers to these questions.

Data Collection Methods

Four main methods of collecting data, as outlined by Eisenhart (1988), were used. These were (a) Participant Observation, (b) Interviewing, (c) Searching for Artifacts, and (d) Researcher Introspection. All four methods were used concurrently during fieldwork, since each method gave a different perspective on the research and each one could be used to inform the other. According to Eisenhart (1988), in ethnographic research, the more perspectives that are represented, the stronger the research design, since each perspective adds to the completeness of the picture one is trying to present. By employing a variety of data collection procedures, the researcher is able to cross-check observations recorded in field notes.

Participant Observation

As Sharp (1989) points out, intensive participation can occur only when the activities involved are credible to the "locals." It is the "locals" who must confer the status of "participant" on the researcher. This is not a status that one can confer upon oneself, a fact that enthusiastic ethnographers usually omit when advocating the method of participant observation. In many circumstances, the researcher is not given this status, and it is then actually long-term observation that provides the data, with participation playing a

secondary, supportive role in the research (Sharp, 1989). With this in mind, I was constantly alert as to how I was being treated by the carpenters and recorded in detail the changes I perceived in their attitudes towards me in the previous section, entitled "Initiation and Acceptance." As mentioned previously, my interpretation of the developments suggests that I became a participant during my third month in the workshop.

Participant observation is seen as the major technique for remaining in the "marginal position of simultaneous insider-outsider" (Hammersley & Atkinson, 1983, p. 100). The researcher has to be simultaneously involved in the activity as an insider and detached enough to allow reflection as an outsider. I found this dual role difficult to play. It became increasingly demanding to wear two hats at once. At first, when everything was fresh and new, it was easy to notice everything from a researcher's point of view. As time went on, and I became more of an "insider," stepping back was sometimes very hard. I had to deliberately force myself to take that step. After five weeks at the workshop, I recorded in my diary:

> I have to persuade myself to get into "researcher mode" at times. It is quite an effort, and I have to talk myself into it. It is so much easier to stay in passive "carpenter mode," casually listening and observing, and getting on with the job at hand. It is far more relaxing for me to spend a day like this, and I sometimes consciously choose to do this if I am tired. I think, "Oh, tomorrow is another day. It is too much of an effort to change the subject to fit in with my 'researcher' agenda. I am rather enjoying this piece of family gossip, or topical discussion, or talk about food!" Or, "I am so enjoying the peace of working with this wood or chair that I don't feel like talking to anyone at all!"

I participated in the workshop as an apprentice, as a helper, as a worker, and in a social capacity. I tried to record my interactions with the carpenters on all these levels and to observe and record their interactions with each other as well. I wrote detailed fieldnotes every evening, sketching the events of each day, thus giving a picture of the day in the workshop. There were obviously certain kinds of events that interested me more than others, and I have given a partial list of important categories below. The categories are derived from a combination of the theoretical context that frames my research objectives

9.2 BEST COPY AVAILABLE

(see Chapter 2) and of my practical experience in the field. The list is necessarily incomplete, since I did not spend the day filling preset categories but tried to absorb whatever I could that would enhance my understanding of what I was experiencing. Occasionally I would write myself a note on a scrap of paper during the day, but generally there was no time for this, and I depended on my memory. The most successful strategy was to recreate the day in my mind in minute detail each evening and to record my observations directly on a word processor. Even on days when I initially felt that not much had been observed or accomplished, on reflection, many interesting and relevant observations did emerge when I began to write.

My observations covered many areas. I recorded descriptions of—

1. the woodworking projects in which I was engaged;
2. the woodworking that I learned and how that learning occurred;
3. the teaching methods of Mr. S and some of the other carpenters;
4. problem solving done by the carpenters alone, with me, or in a larger group;
5. the mathematizing demonstrated by the carpenters;
6. the carpenters' ideas about mathematics;
7. the mathematizing I learned and how that learning came about;
8. links to academic mathematics that I could recognize in the contextualized mathematizing I was learning;
9. learning opportunities for the workers;
10. the role and use of tools in the workshop;
11. problems that the carpenters encounter on certain tasks;
12. social interactions between myself and the other workers;
13. the changing attitudes to my presence;
14. social interactions between the other workers;
15. interactions between Peter and the workers.

Interviews

Interviews ranged from (a) informal conversations to (b) semi-structured conversations to (c) formal structured interviews where I had a script that I wanted to follow.

Informal conversations occurred throughout the day while we worked and during breaks. I would sometimes try to maneuver the topic to something that I was interested in hearing more about. Most times this would work, but occasionally I would meet resistance and be unable to engage the carpenters' attention in my preferred subject. Practical field experience showed me that certain topics would often generate discussion of mathematical concepts. The kinds of topics that were fruitful in revealing mathematizing were those that required the carpenter to—

1. explain something to me verbally or nonverbally,
2. take some action on a problematic,
3. engage in reflection,
4. offer some argument for verification,
5. appraise some work critically,
6. work with a more able peer, or act as a more able peer.

To this end, I continually asked questions while we worked, for instance: "Can you explain why?" "How do you know?" "How can you be sure?" "What do you think?" "How should I do this?" "Why does that work?" "Have I done this correctly?" "How could I have done it better?" "How can I correct this mistake?" "Can you help me with this?" "What would happen if we do ____?" "Is there another way to do this?" "How can I be sure this is okay?" In this way I encouraged the carpenters to talk about what they were doing and to comment on my work.

I was always conscious of the fact that we were in a business environment where production was of the utmost importance. There was no time to stand about and talk. Our informal conversations took place while we worked, and there were many interruptions.

Sometimes when the machines were being used, it was impossible to hear anything, and my research agenda for the day had to be modified.

Some of the informal conversations that I found highly significant have been described as "episodes" and are discussed in detail in Chapter 5. These conversations are significant because they are centered around various problematics that arose spontaneously in the workshop. The episodes provide vivid illustrations of the mathematizing that took place as the carpenters actively engaged with their problematics, and reflected on their actions, finally arriving at solutions that satisfied them.

"Semi-structured" conversations were aimed at the collection of life stories of some of the carpenters. I made prior appointments with the carpenters, and we usually sat in my car at lunchtime to escape the noise in the workshop. I began the conversation with some general opening questions about the life of the carpenter and allowed the conversation to develop from there. I asked certain standard questions if these seemed appropriate (for general script, see Appendix A). In this way I gleaned information about their cultural background, their school experience, their contact with and views of academic mathematics, their carpentry training, and their ideas on craftsmanship and expertise. I recorded a history of the workshop by interviewing Peter, who was also able to give some additional background information about the carpenters.

Conducting the "semi-structured" interviews presented some unexpected hurdles at first. After a few weeks of persuasion, Mr. S consented to speak to me about his life while I tape-recorded the conversation. As usual, there was not much privacy or quiet to be found in the workshop, so at lunchtime we took our sandwiches and sat in my car, which was quiet, warm, and comfortable. What followed was most amusing, if unproductive!

We began by talking about Mr. S's childhood and when he started to learn about woodworking from his father. He was clearly a bit uncomfortable about the tape recorder at first. After a while, he became a little less conscious of it and was warming to the subject when the secretary, who could no longer control her curiosity about what we were doing in

the car, appeared at the doorway with a cup of tea for me. I explained that we were taping an interview. Nothing daunted, she said, "Oh, well, then I'll join you in the back seat!" Dolores was a very large woman who wore big flowing gowns in bright colors. Her presence filled up the back of the car, and she had entirely her own ideas about the direction of the conversation. Poor Mr. S's reveries were completely halted and the interview was ruined!

Structured interviews, where I asked specific questions according to a script I had prepared, were conducted with Clive and Jack during my last week at the workshop (for the script, see Appendix B) . Again, I made appointments for the interviews. Clive's interview was conducted at my home, and Jack's interview took place at his own home. Most of the questions that I asked related directly to work and ideas with which I knew they were familiar. I also included a task that was unfamiliar and that they would definitely regard as being "mathematics." The results of these interviews are discussed in detail in Episodes 17, 18, and 20 in Chapter 5.

While informal conversations were written up as part of my daily fieldnotes, all the "semi-structured" conversations and the structured interviews were tape-recorded. Tape-recorded interviews were transcribed as soon as possible after the event and the transcriptions stored on a word processor. In the workshop we often returned to topics previously discussed, which gave me an opportunity to check on my interpretation of what the carpenters had said.

Additional Methods

In addition to the two primary data collection methods described above, two ancillary methods, also considered to be key data sources for ethnographic research, were used. These were the collection of artifacts and researcher introspection.

The collection of artifacts. This method can assist in gaining a broader understanding of the context than can be gained by relying solely on personal experience. Goetz and LeCompte (1984) point out that anything that the community makes and uses

results in artifacts that constitute data indicating people's experiences and knowledge. This may include physical objects like carpentry tools or everyday household articles. The search for artifacts may also involve archival studies, such as the examination of written records, historical records, or graphical materials. Eisenhart (1988) states that any information produced by the participants or others, which is tangible and has bearing on the topic of study, may be useful.

A variety of artifacts was gathered during the present study. On first entering the field, I studied the history of the Cape Muslim people, since this was the group with whom I hoped to work. This information was useful during my conversations with Mr. S (the only Cape Muslim at the workshop), who was well-informed about the history of his people. I collected written calculations and sketches (or made copies of these later, since in the workshop sketches are often made on very large pieces of wood!) that the carpenters made while working or during our conversations and interviews. I took photographs of tools and furniture under construction to illustrate written explanations in my fieldnotes, and made many sketches depicting problem-solving strategies devised by the carpenters. I was given two hand-made wooden tools, used for drawing oval shapes. During my last week at the workshop, I had a video recording made of the men at work. In the video, I briefly introduced each carpenter and then asked him to explain what he was doing, while the camera recorded his actions. Some of the men thoroughly enjoyed giving detailed explanations or demonstrations of their work. The examination of all these artifacts and the ability to share this tangible evidence with others has enriched my understanding of this community of carpenters.

Researcher introspection. Researcher introspection is the practice of keeping a personal diary to record reflections on the research activities (Eisenhart, 1988). This method is peculiar to research of an interpretivist nature.

I kept a regular diary to trace my emerging insights, feelings, and intuitions so as to document the evolving nature of the study. I recorded how I was treated in the factory, what

kind of work I was doing, and what I enjoyed about it. I lamented my failures and made hopeful schemes for more fruitful lines of inquiry in the next stage of the research process. The diary is interesting and useful to read in conjunction with the fieldnotes. It provided a much needed vehicle for reflection. Self-preservation is a very real consideration when planning an extended period of field work. I elaborate on this in the next section.

The Emotional Demands of Ethnographic Research: Some Coping Strategies

As pointed out by Sanday (1983), participant observation demands complete commitment to the task of understanding. In addition to the length of time required to be spent in the field, other demands made on an ethnographer can be emotionally draining. The keeping of meticulously detailed notes on a daily basis and having to be constantly alert and open to conversation day after day, week after week, can become exhausting. The fieldworker uses herself as the principal and most reliable instrument of observation, selection, coordination, and interpretation (Sanday, 1979). One has to cope with moral dilemmas that arise as a result of the complex web of mutual dependencies between researcher and participants. It is essential to maintain the tension between being totally immersed in another reality while still operating from one's own framework as a researcher with colleagues and an academic committee to report back to.

I found it important to develop a network of colleagues outside of the research setting. I was befriended by an anthropologist from the University of Cape Town who invited me to join a weekly seminar with a group of anthropologists. An old friend who is both an educator and a skilled carpenter proved to be another significant resource. I regularly discussed my research with a mathematics educator at the University of Cape Town. Members of my Special Committee at Cornell wrote to me regularly, giving me feedback on the notes that I mailed to Ithaca. This correspondence provided excellent guidance for the duration of my fieldwork. These opportunities to share my perceptions with other interested people helped to keep some balance in my life.

Every two weeks, I took a one-day leave of absence from the field to review my fieldnotes and prepare them to be mailed to my committee members, with a report on my progress. This was essential to allow some breathing space and to encourage reflection on the total process. Manual labor was new to me, and I often felt physically tired and lacking in energy. Occasionally I took a day off simply to rest and sleep.

The difficulties of entrée may in some ways be matched by the problems of withdrawal from the research setting. After months of daily contact, friendships, expectations, and dependencies had developed, and I was aware that I had to handle the termination of my relationships with care and sensitivity.

Exit

During the last few weeks, Clive, Mr. S, and Jack, in particular, made many references to the fact that I would soon be leaving. Clive complained,

> Why are you so late today? On your last days you must come in very early so you can have more time with us! I am already feeling sad that you are leaving us. It is like giving you off (local way of talking about a father "giving away" his daughter in marriage). Your groom has come to fetch you, and you have to cut all your old ties. You will forget about us soon. As soon as you leave us, you'll wonder who these funny guys were.

He was referring to the fact that I would soon be leaving for the U.S. with my American fiancé. In fact, this paternalistic view made my exit much easier than I had expected. My last day at the workshop was like a pre-wedding celebration. They presented us with a thick handmade yellowwood chopping board that each person had signed. During an extra long lunchbreak, we all watched the video that I had made of the carpenters at work. We left in a flurry of good wishes and sage advice for future newlyweds.

Chapter 5

RESULTS AND ANALYSIS:

MATHEMATIZING BY THE CARPENTERS

Introduction

The aim of this chapter is to address the first research objective of the study. Data are described and analyzed to provide evidence that valid mathematizing is embodied in the everyday activities of the group of carpenters with whom I worked.

Chapter 5 is separated into two parts. In the first part of the chapter, the selection of "episodes" as units of analysis is discussed and the criteria for selection are provided. The main participants are briefly reintroduced, and a series of 20 episodes is described.

The second part of this chapter provides a comprehensive analysis of the nature of the mathematical ideas implicit in the activities of the carpenters. The 20 episodes previously described are the units under consideration in this part of the analysis. The development of categories is documented and discussed, and the "formal, systematic and logical procedures" used to generate constructs are described (Goetz & LeCompte, 1984, p. 167).

Part One: The Episodes

The Establishment of Units of Analysis

According to Goetz and LeCompte (1984), the establishment of units of analysis is one of the first major tasks in processing ethnographic data. Ethnographers are required to describe what they observe, to divide observed phenomena into units, and to indicate how units are similar and dissimilar from each other. Eisenhart (1988, p. 107) refers to the definition of "meaningful" units of the material (meaningful to the researcher or to the participants) as an important initial step in the process of ethnographic analysis. Precedents for the selection of certain incidents as vehicles for description and analysis can be found in the work of Erickson (1982, p. 167), who uses the term "encounter" to describe a "bounded setting" on which to focus the investigation of learning in

classrooms. An "encounter" is defined as a social gathering characterized by distinctive foci of attention and meaning and includes face-to-face interactions between people as well as interactions between an individual and a physical object. The point of focusing upon an "encounter" is that one has to find the "crucial sites in everyday life, situations of time, place and action" where the object of research is most likely to occur (Erickson, 1982,
p. 163). Erickson recommends a chronologically accurate, analytic, narrative style of reporting fieldwork.

Criteria for the selection of units of analysis. In the present study, the primary units of analysis are called "episodes." "Episodes" are defined as those instances where the carpenters (individually or in a group) engaged in a cycle of action and reflection on a problematic in an effort to resolve a dispute, to solve a problem, or to give a convincing explanation to a novice. Furthermore, only those instances where the carpenters demonstrated one or more of the six environmental activities, claimed by Bishop (1988a) to be the foundation of all mathematical ideas, will be considered to be "episodes."[7] An exception to this rule will be seen in the inclusion of Episodes 4, 6, and 12. These episodes were chosen to illustrate some of the carpenters' conceptions of mathematics and of their own relationships to mathematics.

The term "episode" was chosen because the incidents form a series of related events during the course of my continuous experience in the workshop, and each incident is a vignette that tells a coherent story in itself. One group of episodes comprises detailed narratives of a problematic or set of problematics engaged in by one or more of the carpenters. Another group of episodes is included to give the reader an idea of the conception of mathematics that prevailed in the workshop and of how the carpenters saw themselves in relation to mathematics. Many of the episodes describe problems that arose spontaneously from activities taking place in the workplace with the carpenters'

[7] A chart showing which of the six environmental activities described by Bishop are represented in each episode can be furnished by the author on request.

mathematizing unfolding as they discussed their problematics and constructed solutions. Sometimes the problem-solving activities were initiated by the questions I posed in my role as novice. To provide the reader with links to the latter part of this chapter, a brief commentary on the mathematical ideas and processes that are implicit in the activities of the carpenters is included with each episode.

The detailed descriptions of encounters with the carpenters as they discuss and reflect on their problematics are intended to give the reader the opportunity to "hear" the discussions before I categorize my conclusions about them. This represents an attempt to let the participants who made this research possible speak for themselves and present their own cases to some extent. I do not want the richness of the human interactions to be reduced merely to a set of abstract findings.

Introduction to the Episodes

The episodes are arranged chronologically, and the date on which each episode took place is given to provide a temporal perspective. Throughout five months of intensive contact with the carpenters, my role in the workshop was that of a learner. My role as researcher was both as an observer and as a participant. As described in Chapter 4, my physical location in the workshop and the nature of my tasks changed gradually over time. According to my interpretation of events, I was accepted by the carpenters as a bona fide worker or "participant" in the methodological sense during the third month of my time at the workshop.

The episodes described in this section took place across the five-month span, with Episode 1 occuring nine days after I arrived at the workshop and the last episode occuring on my last day at the workshop. During the month of June, I was absent from the field for a period of almost three weeks, hence the long gap between Episode 9 and Episode 10. In some of the episodes I am an observer and in others I am both an observer and an active participant.

The main characters in the episodes. The carpenters had had little contact with formal school mathematics. My principal teacher, Mr. S, a man approaching 60 years old, left school and began as a wage-earning carpenter at the age of 13 years. He studied arithmetic at elementary school, but since he had not been to high school, he had not encountered algebra or geometry. Jack, in his late thirties, left school and started work at the age of 16 years, so he had one or two years of junior high school mathematics. Clive, also in his late thirties, left school and began working at the age of 16 years, after two years of high school mathematics. He was the only artisan in the workshop to have attended classes, including two mathematics classes, at a technical college that provides formal training for artisans. Brian, 21 years old, attended high school for one year before becoming a wage earner. Mathematics syllabi for the first two years of high school include elementary algebra and Euclidean geometry. Life history interviews of Clive, Jack, and Brian indicated that their years at high school had been disrupted for various reasons, and it was difficult to assess exactly how involved they had been in their mathematics classes. Mr. S, Clive, Jack, and Brian are the main characters in the episodes that follow.

Episode 1 (March 9th)

Drawing a star design and locating it on a rectanglar box lid

My first "loose" job was the construction of a jewelry box. Jack had made an inlaid star design on a china cabinet, which Mr. S showed me and described as a "square star." See Figure 4.

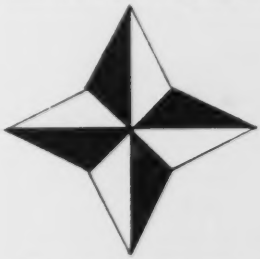


Figure 4. The square star.

Mr. S took the rectangular lid of my jewelry box, about 12 inches by 7 inches, and drew the diagonals in pencil, saying that I should inlay a star design "right in the center" of the lid, where the lines that he drew crossed each other. I took the lid over to Jack and asked him to draw a star similar to the one on the china cabinet but to make the horizontal points of the star longer than the vertical points. "That will look stupid!" he muttered. Using his square rule as a straight edge, he measured the diagonals that Mr. S had drawn. Apparently satisfied with these measurements (the lengths were equal), he drew a small cross at the point of intersection of the diagonals. See Figure 5.

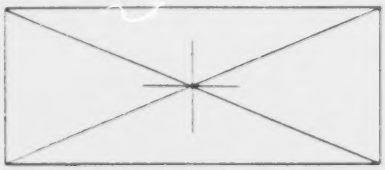


Figure 5. Positioning the square star on the jewelry box lid.

He began to explain how to draw a "square" star (see Appendix C). Finally, at my insistence, he drew the star according to my request (see Figure 6), explaining each of his actions out loud (see Appendix D).

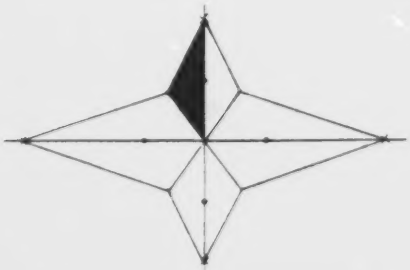


Figure 6. Jack's modification of the square star.

It was clearly the first time that Jack had modified the shape of the "square" star, and he improvised a set of rules for the new shape (see Appendix D). He said that it had to be done that way "to make it right." He claimed that he had learned to draw a "square" star simply by looking at the design on other furniture. In a discussion I had with Peter later that day, he informed me that he had shown Jack how to draw the square star the previous day! This makes Jack's initial resistance to modifying the shape of the star more understandable and the modification more interesting.

Episode 2 (March 13th)

How to find the center of the box lid

Peter and Mr. S looked at the star design and pronounced it to be "too heavy" for the size of the lid. I consulted with Jack, and we decided to make a square design after all. Jack obligingly sanded off all the pencil marks, drew the diagonals in pencil on the lid, and marked the cross on which the measurements for the star shape would be made. I asked Jack why this way of finding the center worked. He looked at me in surprise and said, "Because it is the center!" I persisted, "But how do you know that?"

Gesturing to the cupboard he was making, Jack explained that when he wants to square something, like a cupboard or the frame of a table, he measures across the frame from corner to corner and notes this length. He then measures across the frame the other way, from corner to corner. One length may be slightly longer than the other one. If this is so, then the cupboard or table frame is not square, and this has to be rectified. Gesturing to my block, he said that he had drawn the lines from corner to corner a few days ago and had measured them and found that they were equal; thus the top was square. Therefore he could be sure that the place where the lines cut was the center of the lid.

In fact, his usual practice was to place a long narrow plank across the frame (e.g., of a box or a table), from the bottom left-hand corner to the top right-hand corner, and to make a pencil mark on the narrow plank to indicate the length from corner to corner. See Figure 7.

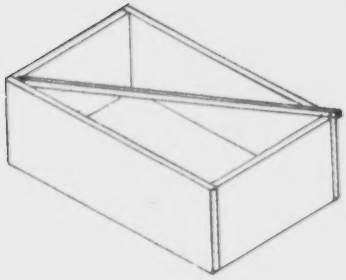


Figure 7. "Squaring" a box.

After this, he would place the same plank in position from top left-hand corner to bottom right corner and indicate this length with a pencil mark on the plank. If the marks coincided, then the box would be considered to be square. If the pencil marks did not coincide, the plank would be cut off at the point midway between the two marks. The plank would then be wedged into the position of the shorter diagonal after glue had been applied to all the joints. Clamps would usually be used to help hold the frame in position while the glue dried.

Jack's explanation used the idea that underlies the familiar physical action of using a narrow plank to wedge diagonally across the frame of a box under construction to ensure that the frame remains square. This was an action that he performed as a matter of routine on a daily basis while making table frames. His actions were clearly not performed in a rote, meaningless fashion, since he was able to use a different representation of this idea, that is, drawing from corner to corner with a pencil on the lid and then measuring and comparing the lengths of these lines to decide whether or not the lid was square.

Although Jack's explanation hinges also on the fact that the sides of the lid should be equivalent, he did not make this part of his argument. This is perhaps because of the way in which the saws are set and the way in which the boards are flipped over during the cutting process. This ensures that if the angles are square, then the sides will be equivalent.

Jack used a concept of "center" that led to the choice of a point that had important symmetries. Straight lines drawn through the point divided the lid into balanced portions.

This encounter is a good example of the kind of explanation I became used to hearing in response to my questions. Physical and tangible examples were given, and explanations usually involved action of some sort. Jack had shown me how to square a table on a previous occasion and therefore knew that I was familiar with the practice.

Episode 3 (March 13th)

The star is crooked and two square rules are different

Mr. S was watching us while Jack used his square rule as a straightedge to draw the set of axes and to mark off lengths along these axes, explaining each step to me in a loud voice. Clive and Jimmy came over to watch. After some struggle, the "square" star shape was drawn in pencil. It did not look quite right to me, and I mentioned this to Jack. I noticed that the triangles were not all congruent as I knew they should be, but was puzzled by the practical implication of this and did not voice this thought. As we all studied the drawing of the star, it was clear that Jack and Mr. S were also puzzled.

Jack acted first. He took some measurements with the straightedge and then said, "The star is not straight, that is the problem—I can use my square rule to check." Mr. S had already gone to fetch his own square rule, having quietly figured out the problem. Jack checked the position of the star with his square rule, using the edges of the lid as frames of reference. Mr. S did the same using his square rule. Sure enough, the north-south orientation of the star was crooked. It was duly erased, but more important for this discussion, Mr. S found that his square rule did not match Jack's exactly!

Jack put the two tools on top of each other (a direct method of checking for congruence) and could feel with his fingertips that the square rules were not at the same angle. He held them up, and it was clear to see that they did not match exactly. Mr. S started to claim that his tool was more reliable than Jack's, but Jack was quick to point out that if his were not true, then all the furniture he constructed would be off-square. He held both of

the squares aloft and said, "These are both square rules and they are different—you can't tell which one is true! Why do you want to believe his?" I agreed to settle for Jack's square rule, since the new sketch of the star, which he had oriented using his square rule, looked good to me.

Episode 4 (March 15th)

Jack expresses some opinions about mathematics

About two weeks after I had started work in the workshop, Jack and I had a conversation that revealed some of his ideas about mathematics. He asked, "Now, Wendy, what are you really doing here? What are you going to do when you leave us?" I said, "I am going to write a thesis about everything that I am learning here." "And then what are you going to do?" he persisted. "Then I am going to teach maths," I replied. A look of astonishment crossed his face. "But then you must be very clever!" he exclaimed. "Why do you say this?" I asked. "Because people who do maths are all ..." and he grimaced and rolled his eyes upwards in a rather graphic parody of someone who is brilliant, but looks idiotic and is socially incompetent! I inquired, "Don't you do any maths in this workshop?" The response was quick. "Oh no! Maybe a bit of adding and multiplying, but that's not real maths!" and he walked off.

Although it was Jack who actually made these statements, his attitude was a good representation of the general attitude of the carpenters. Mathematics was seen as being outside of their sphere of experience, an activity of more intelligent and educated people than they themselves were.

Episode 5 (April 6th)

When is a square rule square?

Three weeks after the two square rules had been shown to be different from each other, I asked Mr. S how he would decide which square was true in a situation such as the one that arose with Jack. He answered that he would use a spirit level and demonstrated what he meant immediately. Placing the spirit level in a doorway he checked that the floor

was horizontal and the wall vertical. When he was sure that they were, he placed the square rule in the corner at the foot of the doorway and told me to look for any gaps between the wall and the square or the floor and square, where the light might be coming through. There were none; thus he was convinced that his square was true.

This method showed that he drew a useful analogy between the function of the spirit level and the function of the square rule. The spirit level was not designed to check for squareness, and I did not ever see anyone else using it for that purpose. It was usually used in the workshop to check whether a horizontal surface was level (like the flat bed of a thicknessing machine) or whether an upright structure was plumb (like the side of a doorframe). Neither of these usual activities linked "level" to "plumb" (i.e., linked horizontal to vertical) to imply squareness. This suggests that Mr. S did not use his square rule by rote but that he had an active understanding of what its use meant. He described the square rule as consisting of two parts; naming the long straightedge the "plumb" and the shorter straightedge the "level" (see Figure 8). Thus the names he gave to the two parts of the tool were not empty labels but were infused with action and meaning.

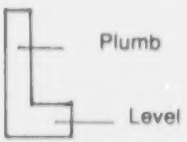


Figure 8. Square rule or carpenters' square.

I asked if he could think of a way to solve the problem without using the spirit level—to imagine that he was on a desert island with two square rules that did not match and that he had to decide which one was true. After some reflection, he generated a second solution that also derived from the way in which he described the components of the square rule. This method was a literal translation and demonstration of the functions of the two parts of the tool.

Pantomiming his actions, he described taking two straight sticks of equal length, putting them into the sand some distance apart and placing a third stick across the top of these two. He demonstrated the action of using a stone hanging from a string to check that the two supporting sticks were standing straight up (i.e., vertically). By miming the action of using the length of string, he indicated that it was important to make sure that the distance between the ground and the tops of the two sticks was identical. Then, he claimed, the shape made by the top crossbar and one of the sticks was exactly what a square rule should look like. See Figure 9.

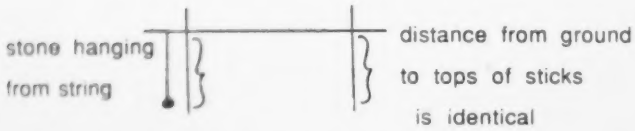


Figure 9. First version of Mr. S's model to check whether his square rule is true.

It was clear that he was assuming the ground to be level. In an effort to challenge this conjecture, and to check whether he would produce a more general solution, I explained that I did not understand exactly what he meant and needed some further explanation. We decided to talk again at lunchtime.

Mr. S brought to my attention his belief that he had actually invented a way to *make* a square rule and not simply to check if any square rule were true. It was clear that he felt a sense of proud ownership of the idea he had described and that designing a square had become his problematic, as will be seen from an exchange that he had with Jack later in the day.

Independently of my interactions with Mr. S, I asked Jack how *he* would check which one of the two different square rules was true. Jack claimed that it would be

impossible to decide between the two square rules unless one had access to something that was known to be true and against which each square could be checked. I said, "But somebody had to invent the square in the first place. Where did it come from?" Jack replied, "Well, where do straight lines come from?" Mr. S entered the conversation, "There is a straight line where the sky meets the sea, at the horizon. It is the straight line that God gave us. You can see if you look at the tops of buildings when you are halfway up the mountain. If you check the tops of buildings, which are straight, you can line them up with your eye along the horizon." Jack looked unconvinced.

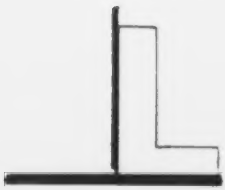
Mr. S's explanation is an indication of how closely his mathematics was integrated into his physical surroundings and the world of his personal experience. His home was built high up on a mountainside, overlooking the ocean, the tall buildings in the city, and the Cape Town harbor.

I asked Jack to think about the problem of the square rule. He queried, "Do you know the answer?" and I replied, "No, I don't know how to solve it, but Mr. S thought of a way. Maybe there are lots of ways. But don't ask Mr. S for his idea! I want to know your own idea!"

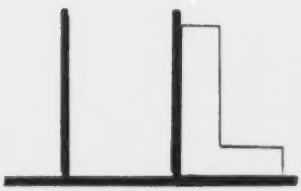
Now and then when Jack passed by my workbench, he said, "Does it have anything to do with circumference?" "Does it have anything to do with a 1-2-3 triangle?" He clearly identified my question as "mathematics" and was searching for the "right" answer to give me, couched in "real mathematical" jargon. About half an hour later, he called, "Hey Wendy! You could do it like this," and he drew Figure 10 in chalk on the workshop floor. I did not think that this would work. In Figure 11, I provide a sketch that demonstrates the flaw in Jack's argument.

I asked Jack to explain why his idea worked. He began to explain, then changed his mind and sketched another solution in pencil on a piece of wood. See Figure 12.

1. Traces line around square rule



2. Flips square rule and traces bottom line



3. Moves square rule to right and traces again

4. Jack claims that the distances between these two lines must be the same

Figure 10. Jack's first attempt to check whether his square rule was true.

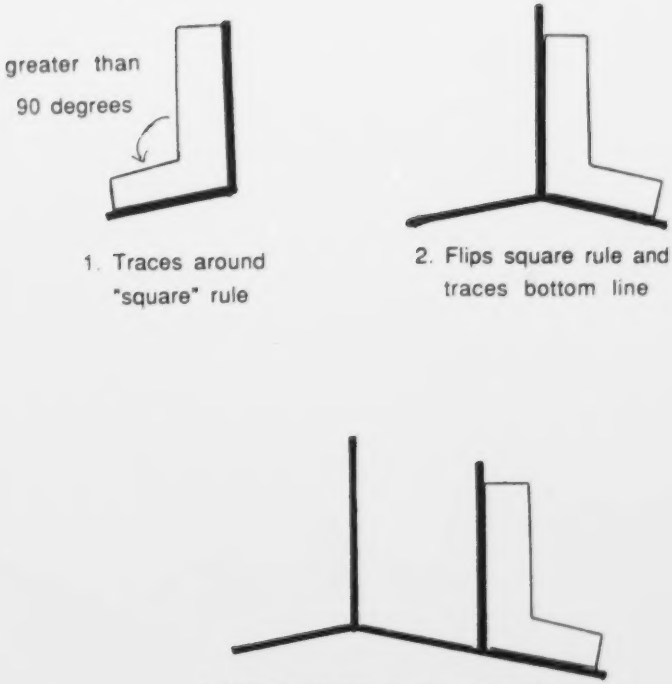
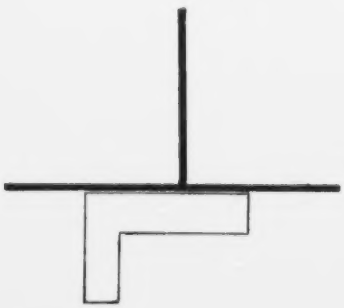

greater than
90 degrees
1. Traces around "square" rule
2. Flips square rule and traces bottom line
3. Moves square to the right and traces again. Jack's claim still holds: these two lines are the same distance apart.

Figure 11. This demonstrates the flaw in Jack's argument.

1. Traces line around square rule

2. Flips square rule and traces bottom line



3. Picks up square and replaces along line Jack claims that this line must be straight

Figure 12. Jack's second (and correct) attempt to check whether his square rule was true.

Although Jack's first solution was flawed, since a square like the one shown in Figure 11 will pass his test (exaggerated to make the effect clearer), he was nevertheless following up on his first idea that a straight line was somehow involved. He tried to use the reflection symmetry of a straight line, making a connection across his ideas. In this case, he realized that he was unable to defend his solution and give a convincing reason as to why it should work, so he carried on looking for an alternative way to solve the problem.

After some reflection and experimental action with the square rule that he had in his hand, he arrived at his second solution, which used the idea of the reflection symmetry of a straight line successfully. He was confident that he had a defensible solution and was able to give me a convincing argument. He drew a diagram to show what would happen if the square were "offsquare" and too wide (i.e., if the angle were greater than 90 degrees). See Figure 13.

Figure 13. Inverted "V" instead of a straight line when the angle is greater than 90 degrees.

He drew another diagram to show what would happen if the square were "offsquare" and too narrow (i.e., if the angle were less than 90 degrees). See Figure 14.

Figure 14. "V" shape instead of a straight line when the angle is less than 90 degrees.

When Mr. S looked at Jack's solution, he remarked, "But you need a square rule to do that!" I reminded him that the problem had begun with two square rules. Paying no

regard to my reminder, he challenged Jack with the remark, "Yes, but how can you make a square if you don't have one?" Jack, however, had solved his own problematic and did not wish to engage in the problematic that Mr S had developed.

At lunchtime Mr. S and I sat at the roadside while he worked with little bits of straw and wood, trying to physically construct his idea in order to explain it to me. Finally, I took two sticks of the same length, put one up on the sidewalk, and one down on the road, to grossly exaggerate the hill I wanted him to take into consideration. He responded immediately, saying, "I will use the horizon to make sure that the top plank is level. There is a straight line where the sky meets the sea. I will take these planks up to the top of a hill, and use the horizon as my natural level."

The whistle blew to signify the end of lunchbreak, and we went back inside. Mr. S began to construct his idea, nailing pieces of wood together to form two uneven legs with a plank across top. He "built up" under one of the legs, using scraps, until "his eye told him" that the top plank was level. The long rectangular window in the far wall served as a mock horizon. As a further way to convince me, he used a spirit level to verify that the top plank was level. Using a piece of string and a castor (a small heavy wheel shaped like a ball), he fashioned a plumb bob and used it to ensure that the legs of the structure were vertical. Then he put his square rule in position and showed me that it was true. See Figure 15.

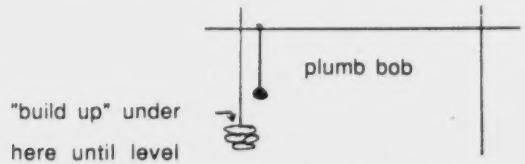


Figure 15. Second version of Mr. S's model to check whether his square rule was true.

His initial assumption that the ground would be level did not stand up to closer scrutiny, and on reflection, the assumption was modified. At all times, his explanation was accompanied by action and he carefully placed his solution in context, describing in detail

how to make the plumb bob and how to use the horizon to check for levelness. Everyday objects lying about on the workshop floor were used to illustrate his ideas. Thus Mr. S's concept of squareness and his acts of cognition as he explored this concept seemed to be deeply embedded in the environment of the workshop and strongly linked to his actions.

As a probe to investigate whether Mr. S would be able to construct another solution, I asked, "Do you think there is a way to do this without using the horizon?" He replied that he could use the 3-4-5 triangle to check whether his square rule was true.[8] Pointing to the square rule, he indicated the side along which he would measure three inches and the side along which he would measure four inches. He drew a line joining the two sides in the air with his finger (i.e., indicating the hypotenuse), saying that the length of this line should be five inches if the square rule were true. It appeared that he had incorporated the idea of the Pythagorean triple as another facet to his concept of squareness. Thus, in a final effort to help me understand how to check that a square rule is indeed square, he gave me an explanation that had its roots in something that I had told him in the past.

Episode 6 (April 7th)

Jack and the "right" answer

I found out from Peter that Jack had asked him how to show that a square is square. Peter had given him some assistance. When I asked Jack about this, he was somewhat

[8] This question revealed what I regard as a methodological blunder on my part. Although it was not my intention to "teach" Mr. S anything, this had occurred inadvertantly at our first meeting, which had taken place six weeks prior to Episode 5. As described in Chapter 4, on the occasion of that meeting he showed me the jewelry boxes he had made for his daughters and explained how he "squared" a small box before gluing it together. In many of the pilot interviews I had conducted with carpenters who worked on building sites, Pythagorean triples had been used to square the corners of buildings. I asked Mr. S if he had ever used the idea of a 3-4-5 triangle to square anything and he appeared never to have heard of the concept. Not realizing that he was to become my teacher for the next five months, I demonstrated how he could square the jewelry box using the idea of a 3-4-5 triangle. We had not talked about the topic since that time.

abashed and replied, "Well, he didn't actually tell me, we worked it out together, and two heads are better than one. That's how they do it at school!" Since Jack's solution clearly evolved while I was watching him, it would not be fair to say that it was not his own idea. However, the fact that he would ask Peter is significant, because it highlights his efforts to find the "right" answer to give me.

Later, when I asked whether he had given Mr. S's problem of how to design a square rule any more thought, he retorted, "You know the *answer* [notice the singular noun] and that is why you keep asking me!" When I protested that there must be many different ways to solve the problem and that I was interested in his ideas, he replied, "Well, you know something about it, and that is why you keep coming back to it!"

His comment was, of course, legitimate, and highlights the difficult path I had to tread as a mathematics educator/novice carpenter. I tried to avoid situations like this because they encouraged Jack to conceal from me the fact that he was unsure of what to do and further convinced him that there existed only one correct answer to my question.

Episode 7 (April 18th)

"Do it by proportion"—The table with the inlaid pattern

Mr. S assigned me to the task of restoring a very old yellowwood table. Several of the original parts were missing, including the table top and the front apron with a central drawer. The table had beautiful inlaid designs on the back and side aprons, made of dark stinkwood. See Figure 16.

Figure 16. The inlay pattern on the back apron of the table.

The first step was to construct a new front apron from a piece of old yellowwood. We began by cutting a plank the same length as the back apron. The back apron was laid on a long plank of the same width, and a pencil line traced along its edge. A saw cut was made along this line. Mr. S took care to show me how to ensure that the back and front aprons were exactly the same length, placing one on top of the other and feeling the ends with his fingertips. This method of checking for congruence is similar to that used by Jack to compare the square rules.

The size of the drawer that had to be replaced was deduced by inspecting the part of the runners that remained on the back apron, and the cutout for the drawer was marked on the new front apron. The inlay pattern on the back and side aprons needed to be scaled down to fit the drawer front. Mr. S instructed me to "do it by proportion," and I began to measure all the relevant dimensions of the *side* apron and to use a calculator to scale these all down proportionally to fit the drawer front. Brian came over to see what was going on, and got involved in the task. He suggested taking more measurements. Soon we had a long list of measurements, and our method began to feel awkward and roundabout. We were not making much progress in drawing the smaller design.

Mr. S returned to see how we were getting along. Ignoring our list of figures, he picked up the back apron of the table and studied it. Using comparisons made initially with his eye and then with his fingers, he estimated that three flower heads could fit along the base triangle of the design. By making another visual comparison, he made a conjecture about the relationship of the length of the back apron to the length of the drawer on which the scaled-down version of the design had to be put. He physically checked the length of the drawer cut from the new front apron by putting it onto the old back apron and confirmed that the drawer was half the length of the apron. Measurements were kept to an absolute minimum; in fact, only the base and height of the triangle were measured.

Mr. S pointed out that the drawer was exactly half the size of the front apron, and the front apron was exactly the same size as the back apron. Therefore we could simply

halve the size of the design on the back apron! Brian began to measure every component of the design on the back apron, including the length of individual petals, the distance from the tip of the upper petal to the tip of the lower petal. In the midst of his measuring, Mr. S stopped him. "The flower shape is made of three identical petals. You just need to cut the template of one petal and turn it around! And look at the grain of the wood on each petal—it runs the length of the petal. You need to cut each one out separately and put three together to make up the flower design." He continued, "You don't have to measure the flower once you have measured the triangle base. It is just necessary to make sure that the sizes agree—three of the flower heads fit along the length of the triangle base in the bigger design, so you want this same thing in the small design." Brian agreed with this, but argued that it did not help him to draw the shape of the petal. Then, picking up a pencil and ignoring all his measurements, he proceeded to draw a perfect flower design freehand by repeating and rotating the single petal! Mr. S checked the proportions using the guidelines he had suggested.

Mr S's solution illustrates his understanding of proportional size and his appreciation of the geometry of the design and of the importance of the internal relationships. His solution avoids the use of measurements and relies on the more direct method of comparison. His solution is simple, direct, and concise and relies on the geometric interrelationships of the piece of furniture. Rather than the fragmentation of design and tasks that Brian and I were trying to perform, Mr. S's practical solution takes cognizance of the broader picture.

Episode 8 (May 15th)

Clive, Mr. S, and the dovetail joints

Building a replacement drawer for the inlay table required the construction of dovetail joints. See Figure 17.

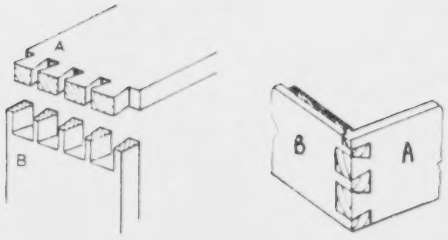


Figure 17. Dovetail joints.

Clive came over to help me sketch the joints in pencil on the drawer components. He brought a sliding bevel, a small metal tool with two moving arms joined by a screw that can be tightened to hold the position of the two arms. See Figure 18.

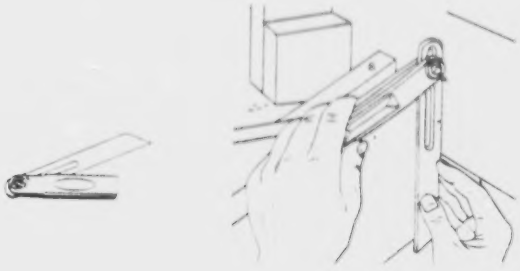


Figure 18. The sliding bevel is used for marking angles other than 90 and 45 degrees.

While explaining that the "angle" of a dovetail joint is "three up and one along" (using an arbitrary unit), he set the angle of the sliding bevel and instructed me to choose the length for the base of the joint and then use the sliding bevel to draw the sides of the joint. Using a trial-and-error method, I succeeded in locating the joints evenly along the back edge of the drawer.

Mr. S had watched me using Clive's sliding bevel with some disapproval and came over to show me his own method of drawing and placing dovetails evenly along an edge. He began by drawing a light pencil line parallel to the edge of the plank along which the dovetails would be placed. The line indicated the depth of the dovetail needed to accommodate the thickness of the plank from which the other half of the joint would be cut. See Figure 19.

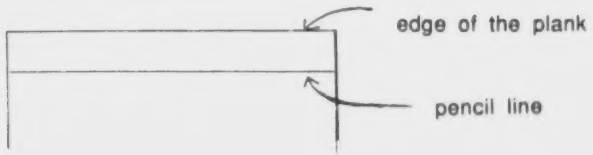


Figure 19. Mr. S's method of preparing to place dovetail joints.

Mr. S's method of placing the joints evenly within these parallel lines was as follows:

Step 1. Find the middle of the plank and draw a line using a square rule. See Figure 20.

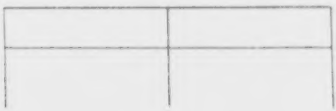


Figure 20. Step 1: Draw a line at the midpoint of the plank.

Step 2. Choose how far "in" from the edge the first joint should start and mark the point. See Figure 21.

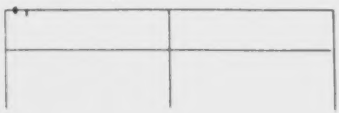


Figure 21. Step 2: Mark a point at a chosen distance from the left edge.

Step 3. Place a point to the left of the line drawn in Step 1, the same distance from that line as the distance chosen in Step 2. Similarly, place a point to the right of the line. See Figure 22.

Figure 22. Step 3: Mark points to the left and right of the center line, the same distance from the center as the first point is from the left edge.

Step 4. Divide the left half of the plank in half again, drawing the center line with a square rule, then repeat Step 3. Do the same with the right half of the plank. Continue these steps until you can join up the points to give a series of even dovetail joints. See Figure 23.

Figure 23. Step 4: Divide the left half and the right half of the plank in half again and repeat step 3.

In this way, Mr. S used a repetitive pattern based on symmetry to ensure the even placement of regularly shaped dovetail joints. His method would result in the dovetails having different "angles" depending on the width of the plank and the thickness of the

wood. He did not, however, indicate any interest in the angle of the dovetail and emphasized instead the even placement of the joints and the choice of a sensible distance from the edge of the plank to ensure a "strong joint."

When I started working on the matching joint cutouts on the second plank (which was to be joined to the first one), Mr. S handed me a "marking gauge," saying, "It does the same as the tape measure and the square—it is like a short cut." See Figure 24.

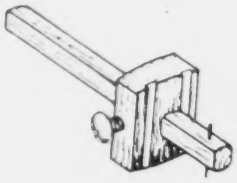


Figure 24. Marking gauge.

He did not name the tool. The small block can move along the shaft, toward or away from the nail at the end of the shaft. The block can be tightened into any position along the shaft using the butterfly nut. The tool is used like a caliper, as a comparison tool. The distance between the block and the nail is set by moving the block and tightening the butterfly nut. This distance may be taken from the thickness of a plank, for instance. When the nut has been tightened, the tool can be pulled along the edge of a plank, with the block pushed firmly against the edge, and the nail will scratch a line parallel to the edge of the plank, at the chosen distance from the edge.

From his comment, I assumed that he meant that using the marking gauge was more direct than taking the two actions of (a) using a tape measure to measure the required length, and (b) using a square rule to "pull" a perpendicular line across a plank. The tool seemed to represent to Mr. S a way of allowing those two actions to be condensed into one direct action.

Episode 9 (May 19th)

Mr. S's ruler

I noticed that although Mr. S usually used his wooden ruler (which is marked in Imperial units), he also owned a metal tape (which is marked in metric units) that he occasionally used. The wooden ruler was broken in half; one side was marked 1–18 inches and the other side was marked 19–36 inches. I asked him if he had any preference for one tool over the other. He replied that he found each one useful for different jobs. Since most of the antique furniture was built according to Imperial units, it made more sense to use the old wooden ruler for restoration work. He said that he might choose to use the tape marked in metric units to take measurements of the new furniture that was constructed in the workshop or to measure the new wood. It was clear from my observations that he favored the Imperial wooden ruler, which was usually kept in the pocket of his coveralls. He used Imperial units for both the jewelry box and the small chest that he helped me to make. He had an intriguing habit of measuring "backwards" from the point marked "36 inches" on his wooden ruler. Despite many previous questions about this practice, it was not until this occasion that he decided to begin to explain his reason to me. He showed me that the ruler is divided into sixteenths of an inch on the side that displays from 19 inches to 36 inches and is divided into quarters of an inch on the side that displays from 1 inch to 18 inches.

Mr. S held up his broken wooden inch rule, remarking, "25 millies is about an inch in the old Imperial measure." He told me that planks used to be sold in thicknesses of $^{7}/_{8}$ of an inch and that it had been very difficult for him to get used to the metric measurements when the metric system was first introduced in South Africa (almost 30 years previously). The person from whom he purchased wood had helped him by telling him that $^{7}/_{8}$ of an inch is about 22 mm. Mr. S had also been told that one whole inch was equivalent to 25 mm, so he reasoned, "Those missing 3 millies are equivalent to the $^{1}/_{8}$ of an inch needed to make up a full inch. So if I look at my ruler on the side that shows 16ths of an inch (half of

1/8), I can see approximately 1 1/2 millies. One half of 1/16 is almost one millie." (In fact, 1/32 = 0.8 mm.)

He clearly wanted a visual way in which to think about the new system of metric units and also wanted to continue to use his old ruler, so he designed a "conversion system" suited to his needs. His system enabled him to "see" the new system of measuring, not to perform numerical calculations and conversions. There are obvious inaccuracies in his conversion system due to rounding errors. Mr. S was seldom called upon to use the metric system during the course of his work, and so the shortcomings of his system had not yet become evident to him. He often impressed upon me the fact that he did not need a tape measure or ruler. Instead, he preferred to use his hand or finger or a piece of wood to compare sizes, as demonstrated in Episode 7.

Episode 10 (July 11th)

The rattan chair

I was given the task of gluing together some loose portions of a couple of old chairs with rattan seats. Realizing that the shape of the seat provided me with a good opportunity to probe for Mr. S's ideas on how to get the center of a shape that was not a rectangle, I asked him how he would find the center of the seat. He did not answer my question directly. He pointed out that the front edge of the frame is longer than the back edge and then proceeded to address the issue of the correct placing of the holes in the seat frame for threading the rattan. See Figure 25.

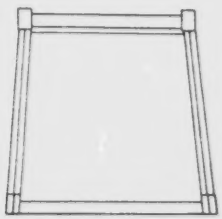


Figure 25. The chair seat frame in which holes are placed for threading rattan.

He explained that the holes would be placed along the front edge of the frame first. This would be done by making the first hole at the midpoint of the front edge and then placing the rest of the holes at equal distances apart along the front edge, using a compass. The holes would then be placed along the back edge of the seat frame. Again, the first hole would be placed at the midpoint of the back edge, and then the same number of holes as on the front edge would be placed at equal distances apart along the back edge. Mr. S emphasized that the number of holes on the front and back edges had to be the same, since the rattan pattern had to remain "square." He pointed out that the holes along the back edge would, of course, be closer together than those along the front edge.

In analyzing Mr. S's actions of finding the midpoints of the opposite sides, I recognized that perhaps he had tried to answer my question about the center of the seat. The line joining the midpoints of the front and back edges split the chair seat into two equivalent halves, so his explanation was consistent with the kind of symmetry he had required for the center of a rectangle.

The placement of holes for the threading of "riempies" (tanned leather thongs), on a chair with a seat similar to that shown in Figure 25, required holes to be made on all four edges of the seat. Mr. S described placing these holes in the same manner as for the rattan seat, finding the midpoint of each side. Unfortunately, I did not ask him about the center of the "riempie" chairseat, so whether or not Mr. S's concept of center is linked to the equivalence of opposite sides remains unclear.

Episode 11 (July 11th)

The Tulbagh chair

A couple of Tulbagh chairs stood next to the chair with the rattan seat. Part of the top of the seat back of one of the Tulbagh chairs, with its distinctive carved shape, had to be replaced. See Figure 26. The right-hand side of the carved back had been damaged.

Figure 26. The back of a Tulbagh chair.

Mr. S gave me a piece of cardboard and a pencil and instructed me to trace around the intact left-hand side of the carving. I asked how this would help us to replace the part that was missing from the other side, and Mr. S responded, "You cut around the lines you have traced, then turn the paper over and draw the shape on the block of wood you want to use. The shape is the same on each side of the middle of the chair. The left side corresponds to the right side."

This strategy is clearly based on an understanding of the concept of symmetry, and of reflection about an axis of symmetry.

Episode 12 (July 13th)

Jack avoids a question

All the stars used for decorations in the workshop had an even number of points, usually four. Mr. S told me that he had seen many star designs with eight and even sixteen points. I decided to try to find out how Mr. S and Jack would produce a star design with an odd number of points.

First I asked Jack to show me how to draw a star with more than four points, and he sketched a design for an eight-pointed star (see Appendix E), with Mr. S looking on. Then I asked him how he would draw a star with five points, or any odd number of points. He looked at me and said, "Have you ever seen such a thing before?" I nodded. "Well, the person who did that must be stupid or something—I think that would look very funny!" With that, he walked away. His reaction to situations like this had by now become familiar, and I was not surprised by his reluctance to become engaged in the question I posed.

Episode 13 (July 17th)

Mr. S draws multipointed stars

A few days later, I asked Mr S how to draw a star with many points. He showed me how he drew an eight-pointed star (see Appendix F) and criticized Jack's way for requiring "all that measuring." Mr. S's method had built-in checks so that the star would "come out right" (i.e., would be symmetrical), which Jack's method did not include.

I also asked Mr. S to show me how to draw a star with five points. He had not done this before but said that he would work it out. He started with a rough diagram. See Figure 27.

Figure 27. Mr. S's first attempt at drawing a five-pointed star.

I asked, "How can I make this neater so that I can use it as a template for an inlay?" He responded by drawing a five-sided figure that looked like a naive picture of a house. See Figure 28.

Figure 28. Looking for a pentagon.

He tried various ways of extending these lines to give a star shape. When this did not give the desired result, he stopped and thought for a bit. Then he drew the diagram reproduced in Figure 29.

Figure 29. Mr S draws a pentagon.

He said, "It will have this shape in the middle and it must have five sides, but each must be the same. Each side will have a point. It looks like a starfish!" He sketched the diagram reproduced in Figure 30.

Figure 30. Mr. S's second attempt at a five-pointed star.

I asked how he would ensure that the sides of the five-sided figure would be the same length, and he replied, "I can use a compass," and sketched a circle freehand round the pentagon he had drawn. See Figure 31.

Figure 31. Mr S suggests using a compass to ensure that each of the five sides will be the same length.

Using his eye, he drew in some pie slices, saying, "I can divide this circle up into five equal parts." See Figure 32.

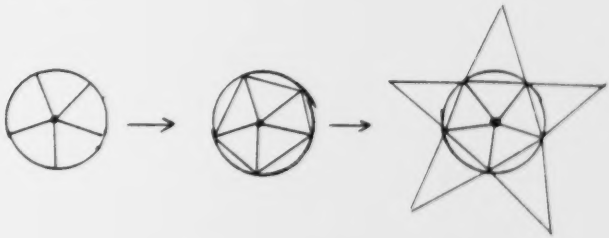


Figure 32. Dividing the circle into five equal parts.

I asked how he would draw a star with nine points, and he began by trying to sketch a nine-sided figure. His drawing kept ending up with eight sides and he commented, "This is an octagon. It is hard to draw one with nine sides." He finally drew the nine-sided figure, then enclosed it in a circle, and put on the nine points to form a star.

He went back to his diagram of the eight-pointed star, which he had drawn at the start of our conversation and drew over the lines again. "You can also use a circle for stars with four and eight points, but a cross is all right—it is easier. But you cannot use a cross for stars with five and seven and nine points. You have to use a circle. So actually, for odd numbers you must use a circle to get it, and for even numbers you can use a cross." (Note that his comment is not altogether correct—crosses work for drawing stars when the number of points required is a multiple of four. For example, a cross would not be helpful for drawing a six-pointed star.)

I had expected Mr. S to begin the nine-pointed star by drawing a circle and marking off nine equal arcs along the circumference. However, he struggled first to draw a nine-sided figure freehand, and then he enclosed it in a circle. It is not clear what function the circle played in his method, nor how he would have used the compass to divide the

circumference of the circle into equal parts. When I asked Clive to draw a star with an odd number of points, he suggested beginning with a circle and using a compass to divide the circumference. He indicated that he would "use his eye" to decide on the appropriate compass span, and then try it out, changing the span if it did not work out the first time. I suspect that Mr. S would also have suggested a trial-and-error method in his use of the compass. Mr. S gradually improved upon his representation of the solution until he reached a point where he articulated a general principle for drawing stars with an odd number of points and stars with an even number of points.

Episode 14 (July 19th)

Replacing a brass escutcheon

Mr. S asked me to replace a brass escutcheon that was missing from the right-hand door of a sideboard (buffet). See Figure 33.

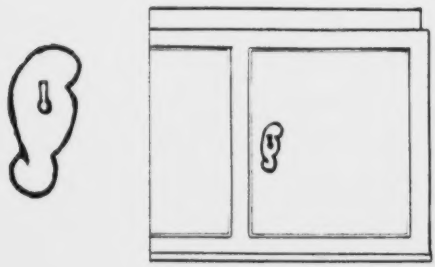


Figure 33. The brass escutcheon on the buffet door.

I asked him to explain how I should begin. Handing me a piece of paper and a blunt pencil, he told me to place the paper over the escutcheon on the left-hand door and to rub

the pencil lead over the paper until the shape was visible. He then instructed me to cut out the shape and glue it onto the piece of brass from which the replacement escutcheon would be cut. Using his hand, with thumb and little finger extended in imitation of the shape to be traced, Mr. S demonstrated how the new escutcheon would be rotated through 180 degrees to match the old one on the left-hand side of the cupboard.

In contrast to the episode about the Tulbagh chair, he did not tell me to turn the template over before tracing around it. The reason for this became evident later, after the new escutcheon had been cut out. The glue with which I had stuck the paper template to the metal proved to be rather resistant to removal. Mr. S told me not to bother about removing the sticky paper, since it would not show once the escutcheon had been glued in place. By not flipping the template, the paper had of course been stuck to the back of the new brass escutcheon.

Episode 15 (July 24th)

Table legs from a plank

Brian was sitting on a big pile of mahogany planks that had recently been unpacked and stacked and appeared to be thinking deeply. I went over to ask what he was doing, and he described his problematic and the solution he had worked out. Brian had used a meter tape to measure the dimensions of a plank. On the plank he had written down the length, width, and thickness and some calculations. He wanted to laminate the wood to form blocks 110 mm square. These blocks would be used to turn table legs. The problematic as he described it was to figure out how many legs he could get from the plank that had dimensions 2.4 m × 0.6 m × 0.075 m, and to come up with a plan of how to cut them from the plank. With obvious pride and satisfaction and by gesturing to the plank while pointing to the calculations he had written down, he explained to me how he had solved his problem. Although he did not draw a diagram, his explanation made clear to me what he had visualized.

As can be seen from my diagram showing his method (see Figure 34), he had envisaged dividing the plank into 15 pieces, each 0.8 m long by 120 mm wide by 0.075 m thick.

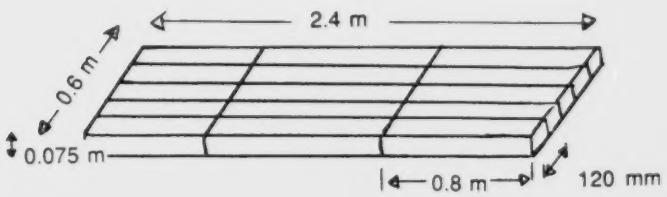


Figure 34. My diagram of what Brian visualized.

He explained his strategy to me by referring to the calculations reproduced below. Since 0.075 m was too thin for the block Brian wanted, the first step in the strategy was to join three of the fifteen pieces (which he had visualized) together. This made five new pieces, each 0.8 m long by 120 mm wide by 225 mm thick, which was about twice the thickness of the desired 110 mm square block. He then imagined cutting these five pieces in half, which would result in ten new pieces, each 0.8 m long by 120 mm wide by 112.5 mm thick. From these he would be able to turn ten table legs according to his original specifications, each 0.8 m long and 110 cm by 110 cm square.

Brian's calculations (performed without a calculator) were as follows:

Step 1. 0.6 m/5 = 120 mm.

Step 2. I can cut 15 pieces from this plank. Each of the pieces will be 0.8 m × 120 mm × .075 m.

Step 3. 0.075 m × 3 = 0.0225 m. This is the same as 225 mm.

Step 4. If I glue 3 pieces together, I will have 5 blocks, each 0.8 m × 120 mm × 225 mm.

Step 5. If I cut each of these 5 blocks in half, I will get 10 blocks, each 0.8 m × 120 mm × 112.5 mm (since 225 mm/2 = 112.5 mm). I want blocks that are 110 cm square. These blocks will be slightly bigger, which is okay because I can cut the extra away when I turn the leg on the lathe.

Brian developed a clear strategy for how to cut the plank and glue the parts to form blocks that he could turn on the lathe. He also figured out how many legs could be cut from a plank of certain dimensions, so he had a way to estimate how many such planks he would need. His solution was based on visualizing the plank in space, imagining cutting the plank into convenient units (i.e., units that divided the plank evenly), and mentally reconstructing new shapes until he reached the desired outcome. In order to perform these mental spatial manipulations, he had to use his knowledge of the metric system and of arithmetic.

Introduction to Episodes 16, 17, and 18

Peter had told me that in his opinion, the carpenters did not understand the concept of volume. He supported his observation by stating that they were unable to calculate the cost of wood, which is priced in cubic meters. I wanted to explore this topic and felt that a good opportunity had presented itself with the conversation I was having with Brian. It seemed clear to me that the solution Brian had mapped out could not have been conceived of without an understanding of the concept of volume. In the next three episodes, involving Brian, Clive, and Jack respectively, I explore the idea that the carpenters had developed an understanding of the concept of volume that was not connected with numbers.

These episodes differ methodologically from the preceding episodes, since the problem did not arise spontaneously from something with which the carpenters were involved. I posed the problem to the carpenters and asked them to solve it, specifically because of Peter's comments about their inability to solve such problems. It was thus clear beforehand that they would have difficulties. I decided to take this methodological risk for several reasons. First, I felt sure that this problem would be seen as a technical matter and something that a novice like me might be curious about. Second, I had formed good

relationships with the carpenters by that time and knew that they felt comfortable with me, and last, since it was just two days before I was due to finish my fieldwork and leave the workshop, if the encounters were awkward or failed in any other way, they could not result in much damage to my data collection.

In the next episode with Brian (from my own point of view), the topic seemed to flow somewhat naturally from what we had been discussing. Although *I* saw the question as a logical progression from our conversation about the table legs, I am sure that he did not perceive it as such. He had not expressed any interest in knowing anything about the cost of the legs before I asked and I was aware of the fact that he would probably have difficulty in answering my question. This made my role as naive questioner difficult to maintain, and the encounter was more like a teaching interview following the form of a Socratic dialogue.

For Clive and Jack, the problem was posed as part of a formal interview that I tape-recorded, so it was even further removed from practice. These interviews were conducted after I had written up the encounter with Brian, so my questions were more circumspect in an effort to avoid reproducing the situation of a teaching interview. I explained to them that Brian and I had tried to solve the problem in the workshop and that I was interested in hearing how they would approach it.

Episode 16 (July 24th)

How much does this plank of mahogany cost?

After Brian had finished describing his strategy for cutting up the plank to make table legs, I asked him if he knew how to estimate the cost of wood. He replied that he did not. I suggested that we try to figure out the cost of the plank from which the table legs would be cut.

Brian looked at the measurements of the length, width, and thickness of the plank that he had written on its rough surface. He added them together : 2.4 m + 0.6 m + 0.075 m and came up with a total of 3.075 cubic meters.

He exclaimed, "Mahogany costs R3000 per cubic meter (over $1,000 per cubic meter). This plank cannot cost more than R9000—there must be a mistake!" He pondered the measurements further and seemed to draw a blank; then he muttered "Now what are square meters and what are cubic meters? There must be three measurements." He looked around the room and suggested that we call Peter to help.

I replied, "Peter looks very busy. Let's try a bit harder ourselves. Maybe it will help if we think about the area of something first. Say we have a room 3 m × 4 m that we want to carpet. How many square meters of carpeting do we need?" I sketched a rectangle and marked one edge 4 m and the other 3 m. See Figure 35.

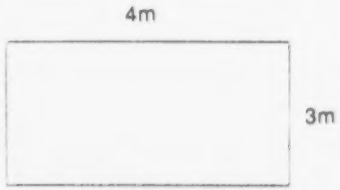


Figure 35. Calculating the area of a rectangular room.

He said, "It must be seven square meters!"

Wendy: "Why?"

Brian: "Oh no, there is another 3 and another 4 (looking at the side opposite to the ones I had labeled), so it must be 7 + 7 =14 square meters."

Wendy: "Can you show me how this works? I don't understand."

He stared at the diagram, then slowly marked off three equal units along one edge, and four equal units along the other, saying "These are each one meter, so I have four along here...." He drew a 3 × 4 grid and counted 12 blocks. "It is 12 square meters!" This visual representation made sense to him and he was clearly satisfied with what he had done. See Figure 36.

Figure 36. Brian's diagram representing area.

Wendy: "And what if the carpet had height or depth? What can we work out from that?"

He turned the rectangle into a three-dimensional shape with a height of three blocks and tried to count the blocks, but he remarked that he could not see all the blocks (see Figure 37). After some thought and counting, he arrived at a total of 12×12 cubic meters.

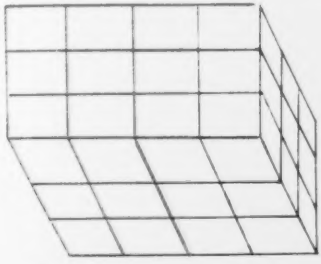


Figure 37. Brian's diagram representing volume.

Wendy: "How did you get that?"

Brian: "Well, there are 12 here (pointing to the top face), and there are 12 here (pointing to the side face). So it makes 12×12. But it sounds like too much. I can't do it. Let's ask Peter to help us."

Wendy: "No, he still seems too busy. I am sure we can do it. Let's figure it out. What about thinking of these as kids' building blocks, and then imagine lifting off the top layer of blocks and putting them aside. How many do you see when you look down on what is left?"

Brian: "Another 12. So that is 24 cubic meters."

Wendy: "And if you had another stack underneath?"

Brian: "Then another 12, so 36 cubic meters! Oh, so you multiply these together. $3 \times 4 \times 2 = 24$ and $3 \times 4 \times 3 = 36$. Yes, that is right. I remember something like this."

Wendy: "So can we think about this plank again? How can we use what you have just done?"

Brian: "We can say the 2.4 m is like the 4, and the 0.6 m is like the 3, and the 0.075 m is like the 2." (Brian multiplied 2.4×0.6, and got 14.4.)

I said, "Isn't that too much?" He did not think so. I persisted, "Isn't 0.6 about like 0.5, which is a half, and what would half of 2.4 be?"

Brian: "It would be 1.2.... But here we are multiplying, so things *must* get bigger!"

Wendy: "But what if I said, 'Give me half of the legs you have turned by teatime.' And you had turned 20. How many would you give me?"

Brian: "I'd give you 10 legs."

Wendy: "And if I said, 'Give me 0.5 of what you have?'"

Brian: "Then again 10."

Wendy: "And if I asked for 0.6 of what you had turned?"

Brian: "Then it would be 11 or 12..., but it is getting smaller because I am giving them away, taking them away! Oh.... Okay, I won't give them away, I'll polish them. If I polish 0.5 of them it will be 10, and if I polish 0.6 it will be 12. So it does get smaller...." He went back to the problem and tried to rewrite it so that it gave him something he believed in, but to no avail.

He pondered a while longer and then remembered a pair of gates that he had recently designed, costed, and built for his brother. He became extremely excited and animated and ran to the office to fetch a calculator. I was conscious that Peter was aware of us, and I felt uncomfortable about the length of time we had spent in discussion, since Brian should have been constructing the table legs. Brian rushed back and drew a diagram of one

of the gates on a plank. As he sketched, he labeled the dimensions of each component of the gate, converting easily from centimeters to meters. See Figure 38.

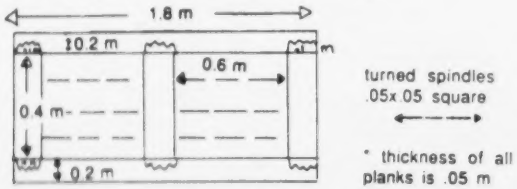


Figure 38. Brian's sketch of the gates he built.

He wrote down three rows of calculations, remembering the answers by rote. The calculation shows the total figures for both gates.

	long	wide	thick	no.	
long top and bottom planks:	1.8	× 0.2	× 0.05 ×	4	= 0.072 m³
upright planks:	0.6	× 0.2	× 0.05 ×	6	= 0.036 m³
spindles:	0.6	× 0.05	× 0.05 ×	12	= 0.018 m³
					0.126 m³

He checked the answers using a calculator, so the position of the decimal point presented no problem. Pointing to the diagram, he described where each measurement came from and exactly how the calculations were set out. I asked if he could use this example to help solve the problem of the cost of the plank. Using the calculator, he tried multiplying the dimensions of the plank again: $2.4 \times 0.6 = 1.44$.

Brian: "So it does get smaller!"

Using the gate problem as a model, he generalized the solution and multiplied the dimensions $2.4 \times 1.6 \times 0.075$, using the calculator. He finally arrived at a price that appealed to his common sense: R324 for the plank.

He said, "Gee, that took a long time for me to teach it to you, but that is because I had to teach it to myself first!"

This episode differs significantly from the previous one. In Episode 16, Brian had a plan that he designed and executed. The gate example, which he used as a model in this episode, also suggests that he had designed and carried out a successful strategy. In this episode, however, it is clear that I played an active role in the solution with which Brian was finally satisfied and that my questioning led him to follow my plan. Interestingly, as can be seen from his last comment, Brian did not appear to be overly aware of being led by my comments. During this episode, Brian made two appeals to Peter as an outside authority and appealed also to the authority of the calculator to deal with the decimal point. In the previous episode, he had discussed his solution with confidence and without appealing to any external authority.

This encounter gives a good indication of the extent of Brian's lack of familiarity with basic arithmetic conventions and algorithms. However, by relying on other strengths like his use of spatial visualizing and by checking his solution against his commonsense knowledge, he engaged in repeated action and reflection on his problematic and finally arrived at an answer that satisfied him. A key part of his success was the use of a previously solved problematic, which he used as a model to solve the novel problem.

Episode 17 (July 25th)

Clive designs a strategy for calculating the cost of a plank

I posed the question to Clive during a lunchbreak, asking "If mahogany costs R3000 per cubic meter, what is the price of a plank 2.4 meters long, by 0.6 meters wide, by 0.075 meters thick?" I had given him a pencil and a piece of paper on which to work.

Clive's initial response was that he had no idea how to estimate the cost of a piece of wood and that only people with specialized knowledge could do such a calculation. He suggested that I should ask Peter if I really wanted to know how much the plank would cost. Clive claimed that he did not know what a cubic meter looked like and made no attempt to explore the idea. I persisted as I had with Brian and drew a rectanglar room, labeling the sides 3 meters and 4 meters, and asked if he could calculate the floor area of the room I had

sketched. He was reluctant, but he made an attempt, adding up the lengths of the sides and arriving at the answer of 7 meters. Clearly he realized that this was not correct, but was not sure about how to calculate the area. I reminded him that he had expertly estimated how many planks I needed to purchase to replace parts of the floor of my house that had been damaged (he had replaced part of the floor of my home one weekend), and he responded with an embarrassed smile. After some more thought, he calculated the area to be 12 square meters and then became involved in trying to sketch a diagram that represented one square meter. He became quite interested in his problematic, but unfortunately the whistle blew signaling the end of lunchtime and we had to go back to work.

Two days later, I asked him the same question, at a formal interview at my home after work.

Clive: "Oh, okay, I don't know...let me see now...for argument's sake, say it is R160 per meter, per cubic meter."

Wendy: "That is very cheap!"

Clive: "Then you will have twelve of two...four, (thinking out loud)...let us say 150 wide and...(pause) 25 thick...so that means you will have twelve of these...." In his mind's eye he had visualized the scheme shown in Figure 39. He did not draw this diagram, but I was able to construct it from his explanation of his solution. From the diagram, it can be seen that he converted all the measurements to millimeters and then visualized dividing the plank into 12 new planks, each 2400 mm by 150 mm by 25 mm.

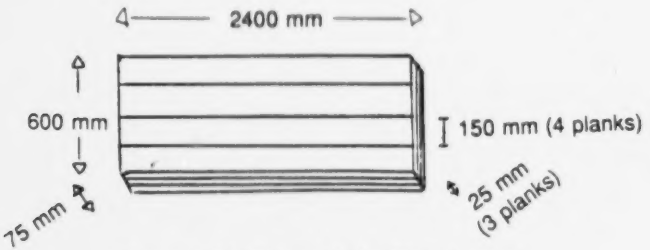


Figure 39. My diagram of what Clive visualized.

Clive: "So you have to multiply all of these. So 24 times 150 times...but first you must get it into meters, so it will be nought point two four, times point nought one five, times point nought two five ($0.24 \times 0.015 \times 0.25$). So you multiply all of these and then once you get that answer, you multiply by twelve, and once you get that answer you multiply by 160. The wood costs R160 per cubic meter."

I pointed to a figure of 2400 he had written down and asked, "Are these millimeters? Two four nought nought millimeters?"

Clive: "Yes, these are millimeters. Yes, it is 2 meters 400, but you bring it to...so it's two point nought four...(he writes 2.40)...yes."

Wendy: "And this is 150 millimeters...or centimeters?"

Clive: "This is 150 millimeters, and this is about 25 millimeters." He puzzled over the conversions to meters that he had previously written down. "This is point one five."

Wendy: "Point one five meters? Why don't you write it all down in millimeters so we can see?"

He wrote down

$$12/2400 \text{ mm} \times 150 \text{ mm} \times 25 \text{ mm}.$$

Clive: "So now I change it to meters." And he wrote

$$2.4 \text{ m} \times 0.15 \text{ m} \times 0.025.$$

He looked at it for a while, then changed it to

$$2.4 \text{ m} \times 0.015 \text{ m} \times 0.0025 \text{ m}.$$

After studying this and murmuring quietly, he finally changed it back to

$$2.4 \text{ m} \times 0.15\text{m x } 0.025 \text{ m},$$

saying, "So you multiply 2.4 by 0.15 by 0.025 and get your answer. Then multiply that by 12. Then once again multiply by 160. And then of course you put the decimal in and that will give you what the whole thing will cost." He did not explain why he changed the rand amount per cubic meter from R3000 per cubic meter to R160 per cubic meter.

Clive was somewhat unsure about place value when converting from millimeters to meters. However, he had no problem converting from meters to millimeters when he began his solution. The problem was posed using meters. He told me that after I had first asked him how to calculate the cost of the plank, he had discussed my question with Peter. Peter had obviously given him the standard algorithm for calculating volume, and they had also discussed visualizing a cubic meter of wood as being a pile of timber one meter high by one meter long by one meter wide. Clive incorporated the standard algorithm for calculating volume into his own strategy of dividing the plank into components that he could visualize. He imagined cutting the plank into smaller units, about the size of the planks with which he usually worked.

Clive commented on the help that Peter had given him: "It made sense in a certain way, but I haven't got his brain! He is a civil engineer and is used to working with figures. He has been (involved) his whole life with quantities. Here we get only the simple things... we do not work out the materials...we do not work out the cost."

As in the case of Brian, Clive appealed to Peter as the external authority when asked to solve the problem. Cost estimation was seen as requiring special skills and knowledge that he did not possess. His actions and reflection on the problematic were constrained by uncertainty, lack of confidence, and unfamiliarity with standard mathematical conventions.

Episode 18 (July 26th)

Jack invents a convenient unit to help solve an unfamiliar problem

The first time I asked Jack how to calculate the cost of a piece of wood was during a lunchtime interview. We were sitting on a pile of planks in the workshop, and I had given him a pencil and a piece of paper for diagrams and calculations. I asked, "If mahogany costs R3000 per cubic meter, what is the price of a plank 4 m long by 0.6 m wide by 40 mm thick?"

Jack claimed that he had no experience with this type of work. He emphasized that only Peter did these costing calculations in the workshop. However, he enjoyed puzzling

over the problem and studied the dimensions of the plank that he had written down. After a few minutes, he wrote 5/4m × 150 × 40, saying, "From that one plank, you can get 4 planks, each 4 meters long by 150 wide by 40 thick." At my request, he drew a labelled diagram of one of the narrower planks he had envisaged. See Figure 40.

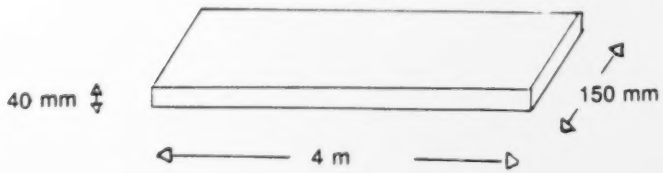


Figure 40. Jack's diagram of the narrower plank he visualized.

After drawing the diagram, Jack reiterated that he did not know how to calculate the cost of wood. Again, I introduced the topic of area and drew a rectangular diagram with sides marked 9 m and 3 m. Jack, using the same incorrect method as Clive, calculated the area to be 12 square meters. He then changed his mind (like Brian), added together the lengths of all the sides, and gave an area of 24 square meters. Finally, he abandoned the 9 m by 3 m rectangular diagram I had sketched, and sketched a 3 × 4 grid, which he extended to a three-dimensional block shape. See Figure 41.

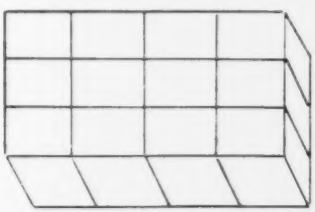


Figure 41. Jack's diagram representing volume.

It was clear that he was trying to extend his idea of area to a third dimension. Lunchbreak came to an end and the whistle sounded, so we could not explore further that day.

The next evening, I interviewed Jack again at his home and asked the same question about the cost of the plank that I had posed to Clive and to Brian: "If mahogany costs R3000 per cubic meter, what is the price of a plank 2.4 meters long by 0.6 meters wide by 0.075 meters thick?"

Jack said that he had thought about this kind of problem since the previous day and had an idea of how to solve it. Using the pencil and paper I had given him, he started performing all sorts of calculations, adding together all the measurements, multiplying by three, seemingly at random. He was clearly uncomfortable and went rapidly from one attempted calculation to the next. I stopped him and asked what was going on. He said that he had asked a friend how to do the problem after I had asked him at lunchtime and that he was trying very hard to remember what his friend had said. He realized that he had forgotten completely. I persuaded him to stop attempting to remember that particular solution but to try to think it out for himself. He finally constructed the following argument that satisfied him.

Jack chose units that allowed him to estimate the cost of mahogany from memory. He told me that standard planks are about 23 mm thick. He visualized dividing the 2.4 meters × 0.6 meters × 0.075 meters plank into a stack of three thinner pieces of 25 mm each. Figure 42 shows my diagram of the scheme visualized by Jack (he did not draw a diagram).

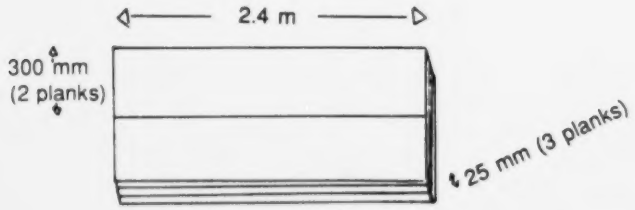


Figure 42. My diagram of what Jack visualized.

He said that the standard width of a plank is about 300 mm and that a mahogany plank of that width and length and thickness would cost R80.

He wrote down the following calculations:

80
×3
240
×2
R 480 is the cost of the plank.

This is probably an appropriate approximate figure for the cost of mahogany planks bought in a small quantity at a retail store. The figure that Brian worked on was the wholesale price for wood bought in great quantity. Jack gave a solution that made sense in the light of his personal experience as a carpenter who had never bought wood in great bulk. Since he did not know the standard algorithm for calculating the volume of a piece of wood, he had to rely on visualization and common sense to construct a solution to the problem that I posed.

Comments on Episodes 17 and 18

The problem-solving strategies devised individually by both Clive and Jack depended on the selection of suitable units. Jack visualized cutting the plank into units of which he knew the price, and this simplified his calculations. He selected the unit because he knew its price. It is not clear what he would have done had the plank not split fairly neatly into an integral number of his selected unit. Clive chose a unit that he could visualize and that was similar to the size of plank he was accustomed to handling. He imagined dividing the plank into twelve of these units and then applied the standard algorithm to the unit and multiplied by twelve.

On analyzing their responses in more detail, I conclude that the difficulty lies not with a misunderstanding of the concept of cubic measure but with (a) a misplaced sense of inferiority—a feeling that only certain people are qualified to do calculations like this, that this falls beyond their sphere of expertise; (b) a confusion about decimal places, converting between millimeters and meters; (c) a lack of experience with mathematical conventions;

and (d) the fact that a cubic meter is too big a measure to be useful to a man who builds tables and chairs. The amount of wood needed for a chair is always some minute fraction of a cubic meter that is very difficult to picture. As Peter pointed out, cubic feet are so much easier to visualize and to talk about. One needs roughly one cubic foot of wood to make a chair, and three cubic feet to make a table.

Although these episodes seem to confirm that the carpenters have developed a visual understanding of the concept of volume, the episodes also served to uncover many misconceptions and to highlight the severe limitations that are imposed on the abilities of the carpenters to perform simple numerical calculations. The demeanor both Jack and Clive displayed while they were trying to solve this problem was totally different from their usual confident manner. They stumbled and struggled and made it clear to me that this calculation was not in their domain of work—it was a special calculation for which Peter was responsible. It was seen as "mathematics" and therefore beyond their reach. In the past, Jack had purposely avoided any situations with me where he might not be able to answer a question. This time, however, he was trapped in an interview situation and was clearly somewhat uncomfortable at times.

Episode 19 (July 26th)

Why isn't "straight up" a full 100 degrees?

Jack had used his square rule on countless occasions to help me check that joints were square, to "pull" a line along the edge of a plank to be cut to ensure that the cut would be square. In his day-to-day work constructing tables, the square rule was often used to check for "squareness" along the way. He had constructed an innovative solution to the question "When is a square rule square?" described in Episode 5. I was therefore surprised by this exchange that took place during a formal interview with him at his home.

I had asked him, "What does it mean to be square?" The following excerpt is taken from his discussion:

Jack: "Now you get another square...it's not a square actually, but you can make it into a square. It is called a sliding bevel. It lets you cut in any angle you want." See Figure 18 (page 114).

Wendy: "That is what Clive uses for drawing dovetail joints."

Jack: "Yes, you can use it on a dovetail joint. It can reach from nought to a 90-degree angle."

Wendy: "So nought would give you what...just a straight edge?"

Jack: "Yes, just a straight line."

Wendy: "And a 90-degree angle would give you?"

Jack: "A 90-degree angle will give you that" (he drew an angle of about 80 degrees). See Figure 43.

Figure 43. Jack called this a "90-degree angle."

Jack: "This is 100 degrees...(he drew a 90-degree angle)...and a 90-degree angle will give you that" (he drew over the line at about 80 degrees that he had already drawn). See Figure 44.

Figure 44. Jack called this a "100-degree angle."

Wendy: "Which is 100 degrees? The line going straight up?"

Jack: "The line straight up is 100 degrees. People will say that this is a 90-degree angle (pointing to the perpendicular line),...but I do not call this a 90-degree angle."

Wendy: "Don't you? Why not?"

Jack: (hesitating) "Umm...then I am out of square...I don't know."

Wendy: "Your square rule, the metal tool, is that 100 degrees, or 90 degrees?"

Jack: "It's a 90-degree angle...Yes...because they say the 45 is halfway. Split this in half. They say that's 45 degrees and that's 45 degrees. I want to know why you don't get 100 degrees."

Wendy: "When you add them up?"

Jack: "Yes, when you add up the square. Square means "straight in line." That is on its full amount. When it reaches its full point, the maximum point...what is the degrees? This is the sign for degrees, isn't it?" (He drew a small circle.)

Wendy: "Yes, that's the right sign. And that (pointing to the vertical line) is 90 degrees."

Jack: "But why is it 90? Why is it not a full hundred?"

Wendy: "Mmmm...Are you asking me?"

Jack: "I am trying to figure it out myself...that's what I am trying to figure out. Basically it starts from nought to 90 degrees angle. On your machine (the table saw) you always see nought to 90 degrees angle. The 90 degrees angle is the full angle, the full size."

Wendy: "What if this went all the way down. What if you bent this all the way down, then it would be...." See Figure 45.

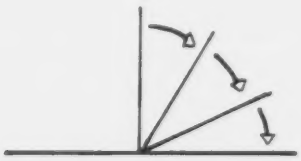


Figure 45. Moving the vertical arm clockwise through 90° ("bending it all the way down").

Jack: "If you bent this all the way down..."

Wendy: "All the way down until it was level again."

Jack: "That will be called a nought. Yes, that is nought. You start from nought...to nought." See Figure 46.

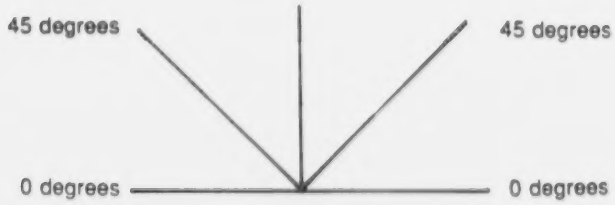


Figure 46. My diagram of Jack's description of angular measure.

Wendy: "With a 90 in the middle."

Jack: "That will be your 45 on your right side and that will be your 45 on your left side. And all the other degrees, 30 or whatever, 70 come in between here."

Wendy: "So it looks like you never get to 100?"

Jack: "Yes, I want to know why is it that you never get to 100!"

In combination with the previous two episodes, this conversation with Jack forced an examination of my ideas about what it means to understand a concept. There appears to be a difference between understanding or knowing the conventions that surround a concept (these conventions are very seldom, if ever, questioned) and understanding a concept in a way that allows its active, practical use. Jack understood the concept of measuring angles, knew what a right angle looked like, and knew that a right angle was said to measure 90 degrees. He used the concept of "squareness" and the properties of a rectangle successfully in his work. However, he was not familiar with the conventions of angle measure, that it was agreed that there would be 360 degrees in a circle, that an angle of 180 degrees represents a straight line. On the table saw that Jack used, degrees were marked on a dial as shown in Figure 47.

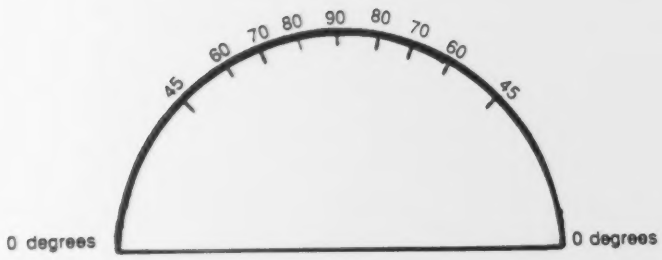


Figure 47. The dial on the tablesaw in the workshop.

There was no evidence of any angle that might be assigned a number greater than 90 degrees.

As in the case with the carpenters' understanding of volume, Jack seemed to have an understanding of angles, straightness, and squareness that was rich in spatial awareness but poor in its link to numerical values.

Episode 20 (July 26th)

Clive and Jack talk about straightness

The following are excerpts from formal interviews that I conducted with Clive and Jack, respectively. I had asked Clive, "When do you call a line straight? How do you check that something is straight?"

Clive: "Okay, you get a piece of timber. First thing you do is pick it up and look at the edge of it or the side of it, or whatever you want to check is crooked or not. When you look down a piece of timber and check it with your eye, the moment you can see a dip, you know it is not straight. You can also check it by putting a straightedge along it and looking for any gaps. Or you can lay the piece of timber on a flat surface and check for gaps. Another way to check it is using a 'try square' (see Figure 48), putting the try square on a flat edge and seeing if it is opening at one end or at the other. Or with surface planing or by hand planing."

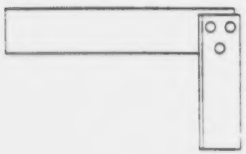


Figure 48. Try square: The wooden handle sits flush against the edge of the surface, and the metal blade indicates a 90-degree angle.

Wendy: "How would you use the try square to check if a piece of timber is straight?"

Clive: "If you want to check a wide surface, you put the try square against the edge so that it rests on the wide surface. Say, for argument's sake, that your timber is 3 inches thick and 20 inches wide. Then you put your try square resting on the 20 inches wide, and hard up against the edge. If you see a gap between your timber and your square, then you know your timber is off. It is not straight." See Figure 49.

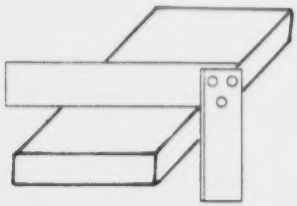


Figure 49. Using a try square.

Wendy: "How would you use surface planing or hand planing?"

Clive: "Say, for instance, you have a piece of rough timber that you want to straighten out and you are in a joiner's shop or any other machine shop. You surface the widest section first with a surface planer and get that straight. Each surface planer has a long flat surface, and you can always check the timber on that by laying it flat down on there and looking for gaps. Or you can check it by eye. When your flat surface is finished, then you turn it and put the flat surface against the fence of your surface planer and plane the bottom

edge of the plank. By doing that you will get your two edges 90 degrees. You first make the surface straight, and then the edge."

Wendy: "Okay, I see that now. How would you check that a round dowel is straight?"

Clive: "You can never get a dowel perfectly straight. You can look down the length of a dowel, and when you can see it go up and down, up and down, and on the side you can see in and out, in and out, then it is not straight. You can also turn it and look down the length of the dowel. You will see one point and everytime you turn it, you miss one point. It seems to dip away and you can't see it, then all of a sudden you see it again. When a dowel is made, the wood goes through a machine and it goes through so fast, turning at the same time. When it comes out of the machine it is hot! It has to cool down and it twists and turns. If you do get it straight, it's just luck."

Clive's practices are based on an implicit understanding of straightness. He imagines taking a piece of timber and straightening it to form a smooth plank, thus contextualizing my question and turning it into a familiar problematic. His methods and explanations are based on actions that originate from an underlying foundation of the symmetries of a straight line.

I asked Jack similar questions in his interview: "Jack, when would you say that a line is straight, and how do you check that something is straight?"

Jack: "I could use my square to check, but if I haven't got that, I could use my eye. Say this is a piece of plank here (he makes a horizontal plane using a piece of paper, holds it up at eye level, and squints along its top surface). Once you see the front part, you have to see the back part of the wood. If you don't see that, then you know that there is a hump somewhere, or a dip. Your eye falls right into that dip or hump. If your eye falls on nothing, then your eye will fall right on the end of that piece of wood and then you will see that it is straight."

"To make your wood straight, you can plane it on a planing machine. Say you have a rough plank that is bumpy, you have to put it through the surface planer to get that side smooth. Then you take it to the thicknesser, and set the machine to any thickness you want. You put the side that you just planed onto the thicknesser table, on that flat surface, and push it through the machine."

Wendy: "How does the thicknesser machine work?"

Jack: "The thicknesser has a flat table on which you rest a plank. There are blades above the table. You can lift the table up (closer to the blades) or down (further from the blades) by turning a handle. Both of the gears have to shift evenly to keep the balance of the table even. The other day it was 'out.' I measured one side of my plank and it read 23.5 mm, and the other side read 22 mm."

Wendy: "How would you know if a round dowel, like a curtain rod, was straight?"

Jack: "A round dowel? I would roll it onto a smooth surface and if it is 'out,' it will flop around. You just push it along, and as it rolls, you can see where it is high and where it is low."

Like Clive, Jack imagined himself handling a plank of wood, solving a problematic that occurs frequently in the workshop. He set my question in context. He also used the method of "sighting" along the plank, which depends on the property of light traveling in straight lines. He described using the planing machine to straighten one face of the plank, visualizing parallel horizontal planes. The use of the thicknesser extends the idea of horizontal planes. He noted the importance of the levelness of the table to ensure that the planes remained horizontal, pointing out the consequences (uneven thickness of the plank) if the table were not level. His comments about the dowel reveal an implicit understanding of the rotational symmetry of a straight line.

Part Two: Analysis of Episodes

As has been pointed out by Goetz and LeCompte (1984), data collection and data analysis are inextricably linked in ethnography because the ethnographer may not know

exactly what questions to ask and what activities to observe until initial perceptions have been analyzed and tentative conclusions have been formulated. Analysis is thus not relegated entirely to the period following data collection but is a process that continues throughout the research endeavor.

Grouping of Episodes

Following the pattern of the analysis process as laid out by Goetz and LeCompte (1984) and Eisenhart (1988), the next step (after defining the units of analysis) involves grouping the units of analysis according to similar characteristics. An inspection of the data led to the identification of three groups of episodes. All of the episodes, except for Episode 20, took place in the workshop.

Group One. Group One includes those episodes that developed as a result of problematics that arose spontaneously for the carpenters and that were strongly linked to their usual activities. Most of the encounters were encouraged or covertly engineered by my questions as a novice. This group comprises Episodes 1, 2, 3, 5, 7, 8, 9, 10, 11, 13, 14, 15, and 19.

Group Two. Group Two includes those episodes that developed from a problem that I posed and that was less directly related to customary workshop activities. These encounters were in the form of interviews, rather than being prompted by the questions of a novice in the context of the workshop. Three of these episodes took place away from the workshop. This group comprises Episodes 16, 17, 18, and 20.

Group Three. Group Three includes those episodes that highlighted the attitudes and the relationships of the carpenters to their conceptions of mathematics. This group comprises Episodes 4, 6, and 12 and certain parts of Episodes 1, 5, 16, 17, and 18.

Categorization Within Groups of Episodes

Each episode was scanned through systematic content analysis, in order to generate descriptions for the properties and attributes they contained. By comparing, contrasting, and synthesizing these properties and attributes, three categories were finally derived. The

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derivation of the categories was also linked to the facilitation of the primary objective of the present study—to show that the carpenters engage in valid mathematizing. The three categories were—

1. conventional mathematical concepts that were embedded in carpentry practice as demonstrated in the workshop,
2. the carpenters' conceptions of mathematics and feelings about mathematics,
3. significant characteristics of the carpenters' mathematizing.

Each episode was scanned for examples or other evidence of these three categories, and a 20×3 grid was drawn up to record the links between episodes; the frequency of occurence of ideas, practices, and attitudes; and the elements making up the characteristics of the carpenters' mathematizing[9]. In the next three sections, the results of this phase of the analysis process are elaborated for each of the three categories.

Conventional Mathematical Concepts That Were Embedded in Carpentry Practices in the Workshop

The first two groups of episodes were examined to identify any conventional mathematical concepts that may have been implicit in the actions or in the discourse of the carpenters or that may have been explicitly discussed and used in the workshop. Recurrent uses of the same mathematical concepts as well as other common themes were searched for within each group of episodes and across the two groups of episodes. Similarities and differences are pointed out across the two groups of episodes.

Not unexpectedly, there is very little discussion of mathematics for its own sake in the workshop. This does not imply that mathematics is not discussed or used in the workshop; it merely means that conventional mathematical concepts and conventional mathematical terminology are often concealed behind physical activities, hand gestures, silent acts of visualization, and the carpenters' own terminology.

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[9] Tables documenting the analysis of each episode can be furnished by the author on request.

The concept of "straight." In Episode 20, Clive articulated several instances in that his practices show an implicit understanding of the concept of something being straight. He gave me the one example of how he would "look down a piece of timber" (sometimes this practice is called "sighting along a line"), and see "dips" if the plank was not straight. This method works because light travels in straight lines. Three other methods that he suggested for checking whether a piece of timber is straight implicitly used the property that a straight line has reflection symmetry through itself. These three methods were putting a straightedge along a plank, laying a plank on a known flat surface, or putting a try square against the surface of a plank and checking for gaps. Using the surface planer is based on the same property of reflection symmetry.

Clive's first explanation of how to check the straightness of a round dowel was based on "sighting along" the length of the dowel, similar to his first response about the plank (see Episode 20). He also gave another way, based on the property of rotational symmetry possessed by a straight line, describing how to turn the dowel while looking down its length. "You will see one point, and everytime you turn it, you will miss one point. It seems to dip away and you can't see it, then all of a sudden you see it again" (see Episode 20).

In Episode 20, Jack demonstrated the act of "sighting" along the surface of a piece of paper and mentioned the "dips or humps" into which his eye would "fall" if the edge was not straight. His explanation of using the planing machine and the thicknesser was based on the visualization of two horizontal plane surfaces. His response to checking whether a round dowel was straight involved the property of rotational symmetry. In Episode 5, Jack's solution was based on the reflection symmetry of a straight line.

The concept of parallel lines. Jack revealed an understanding of the concept of parallel lines when he used the sides of the box lid as frames of reference in correcting the orientation of the star (see Episode 3). Since he knew that the box lid was rectangular, Jack realized that in order to draw a correctly oriented set of orthogonal axes, he should draw

lines parallel to the sides of the box lid using his square rule. In Episode 20, he visualized using parallel horizontal planes to explain how he would produce a straight, even plank from a rough piece of timber.

In Episode 5, Mr. S used the concept of parallel lines implicitly in his explanations of how to use three straight sticks or three planks to prove that his square rule was true. Based on his first (and faulty) hypothesis that the ground would be level, he claimed that if the distance from the ground to the top plank were the same at both supporting planks, then the top plank would also be level (i.e., then the top plank would be parallel to the level ground). In his more powerful and general version of that solution, he explained how he would "line up" the top plank with the horizon, using the horizon as a natural "level" and again implicitly using the concept of parallel lines.

Parallel lines intersected by a transversal. Parallel lines were again used implicitly by Mr. S in his method of placing dovetail joints evenly along the edge of a drawer (see Episode 8). The repetitive pattern that he employed can be viewed in terms of a series of transversals cutting two parallel lines. Thus he ensured that the "angle" of his dovetail remained the same along the drawer front (since alternate angles are equal). See Figure 50. Note that neither Jack nor Mr. S used the term "parallel" when discussing their actions and explanations during these episodes.

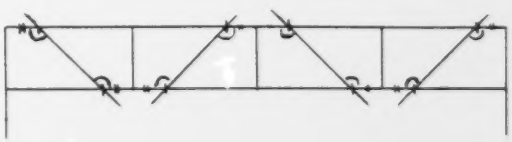

Figure 50. Analyzing Mr. S's method for placing dovetails in terms of parallel lines and transversals.

The concept of "center." As discussed in Episodes 1 and 2, Jack and Mr. S used a concept of center that had important symmetries. Their concept of the center of a

rectangular figure being at the point of intersection of the diagonals of the rectangle depended upon the fact that the opposite sides of a rectangle are equivalent in length. In Episode 10, I tried to probe for Mr. S's concept of the center of a four-sided figure that did not have opposite sides of equivalent lengths. His explanation suggested that his concept of center in this case was tied to symmetry, consistent with his concept for a rectangle, but my interpretation remains unclear.

The concept of symmetry and the axis of symmetry. The concept of symmetry was often implicit in the actions of the carpenters. They would sometimes verbalize their actions while using symmetry but never used that conventional label. Jack used the reflection symmetry of a straight line in his explanation of how he could show that his square was square (see Episode 5). Mr. S used the concept of symmetry when explaining to me how the carved back of a Tulbagh chair should be traced (see Episode 11). He told me to trace half of the design on a piece of paper and then turn the paper over to get the other half of the design. He said, "Both sides must be the same, they correspond" (i.e., he referred to reflecting half of the design about the axis of symmetry of the whole design). Clive made similar comments when using a tool that enabled the user to draw oval shapes (not reported as an episode), "You only have to draw half of the shape, because you can get the other half by folding the paper over."

In Mr. S's system for the even placement of dovetail joints, he began by finding the center of the piece of wood and drawing a line perpendicular to the edge along which the joints will be placed (see Episode 8). He used this line as an axis of symmetry to guide the placement of two points. His system worked by focusing on different sections of the plank and using the axis of symmetry of each successive section to guide the placement of points.

Comparing Mr. S's instructions to me for tracing the shape of the back of the Tulbagh chair (Episode 11) and for tracing the shape of the brass escutcheon (Episode 14) highlights a significant point. In contrast to his directions for the Tulbagh chair, where he specifically told me to turn the template over before tracing the shape onto a piece of wood,

he did not tell me to turn the template for the brass escutcheon over before tracing it. As described previously, by *not* flipping the template, the paper was stuck to the *back* of the new brass escutcheon, leaving the front face unblemished. Since the paper was difficult to remove cleanly, this strategy resulted in the optimum solution. The choice of strategy shows an understanding of the distinction between reflecting a shape through an axis of symmetry in two dimensions and rotating a shape 180 degrees around an axis of symmetry in three dimensions. This distinction remains blurred in formal mathematics—the results of these two actions look identical in a textbook. In practice, however, the distinction is clear, as exemplified by Mr. S's different strategies.

The concept of a right angle. Mr. S revealed his understanding of a "square" angle (i.e., a right angle) as the intersection between that which is "level" and that which is "plumb" (i.e., horizontal and vertical) in Episode 5. In Episode 20, Clive mentioned a similar idea when he described straightening first the face of a plank on a surface planer (i.e., making it a horizontal surface) and then rotating the plank into an upright position with the horizontal surface against the fence of the plane in order to straighten the edge of the plank. He said by doing that, "you would get your two edges to 90 degrees" (i.e., you would create a right angle along the edge of the plank).

The diagonals of a rectangle are equal. The common practice of "squaring" the aprons of a table during construction carries an implicit understanding of the fact that the diagonals of a rectangle are equal (see Episodes 1 and 2). The explanation given by Jack in Episode 2 supports my claim that the practice is based on an active understanding of the concept and is not performed rotely. The terms "rectangle" and "diagonal" were not used by the carpenters. All rectangular shapes were referred to as "square," and carpenters referred to the "length/distance from corner to corner."

The diagonals of a rectangle bisect each other. The practice of drawing the diagonals of a rectangle and using their point of intersection to locate the center of the rectangle is based on the fact that diagonals of a rectangle bisect each other (see Episodes 1

and 2). Again, I claim that Jack demonstrated an active understanding of this concept in his explanation in Episode 2.

The concept of triangle. Mr. S used a 3-4-5 triangle to support his argument that his square rule was square (see Episode 5).

Spatial visualization. In Episode 14, the method that Mr. S instructed me to use for replacing the missing brass escutcheon was based on the concept of symmetry. In this encounter, he used his hand, with thumb and little finger extended to imitate the shape of the escutcheon, to show me how rotating the existing left-hand escutcheon would give the correct shape for the replacement right-hand escutcheon. By mentally reorienting the shape in space, he knew that the template should not be flipped over before gluing it to the metal because this would ensure that the sticky paper would end up on the back of the template and would thus not have to be removed.

Similarly, Jack mentally manipulated (flipped) the imaginary shape of an "off-square" square in Episode 5, when he drew a sketch of what the outcomes would be if a square were "off square and too wide" or "off square and too narrow." In Episode 7, Mr. S mentally manipulated (rotated) a shape in space when he recognized that the flower design on the inlaid table was made up of the rotations of a single petal.

In Episode 15, Brian performed the following mental operations. He visualized the plank from which he wanted to cut the leg components, divided it into sections, re-asssembled these sections into new structures, and divided these again into sections that matched his mental image of the legs he wanted to turn on the lathe. Brian did not make any diagrams while solving the problem. Although included in this section, entitled "Conventional mathematical concepts that were embedded in carpentry practices in the workshop" (see page 151), it is argued that the kind of visualizing that Brian used to obtain the optimum solution goes beyond the kind of visualization usually required by formal mathematics. This point will be discussed further in the section entitled "Constructing functional visual units" (see page 166). In Episode 16, he constructed a visual concept of

volume when he sketched a three-dimensional block shape and imagined moving units of the block aside in order to see and count all the units composing the block.

In Episodes 17 and 18, Clive and Jack each made mental constructions of units in order to solve the problem of calculating the cost of a plank. Clive imagined sectioning the plank into congruent units that were the size he was used to handling, whereas Jack imagined dividing the plank into congruent units of which he knew the price. They visualized the acts of sectioning and counted the images of the smaller units.

The concept of congruence. The concept of congruence was implicit in Jack's placing of his square on top of Mr. S's square to show that they were not the same (see Episode 3) and was also implicit in the customary practice of placing one plank on top of another and feeling that they were the same size without taking any measurements (see Episode 7). The common practice of creating templates by tracing, in order to reproduce shapes, is also based on the concept of congruence (see Episodes 11 and 14). The term "congruence" was not used in the workshop.

The concept of proportion. The concept of "proportion" was specifically mentioned by Mr. S when he instructed me to make an inlay design for a drawer front that had to be similar to but smaller than the design inlaid on other parts of the table (see Episode 7). Mr. S used proportion not only to scale the design on the back apron by one half, he also used proportion to check that the internal relationships of the design remained in harmony with those of the original.

In Episode 9, Mr. S described his method of converting between the metric system and the Imperial system. He said, "Those missing 3 millies are equivalent to the $^{1}/_{8}$ inch needed to make up a full inch." It is clear that he was visualizing and comparing the relative sizes of the two lengths represented by those numbers and that he had concluded that they were equivalent in length. He continued by reasoning that the length represented by $^{1}/_{16}$ inch would be equivalent to the length represented by 1.5 mm, and finally that the length represented by $^{1}/_{32}$ inch would be equivalent to the length represented by "almost one

millie." Thus by using proportional reasoning, he constructed a visual image of the approximate length represented by 1 mm.

The concept of volume. Episodes 15, 16, 17, and 18 support the claim that the carpenters have an understanding of the concept of volume. This understanding is characterized by a strong sense of the spatial and visual meaning of volume rather than by a knowledge of how to calculate volume numerically.

In Episode 15 and in Episode 18, neither Brian nor Jack numerically calculated a volume, but both used the concept of volume visually to section planks into units that were useful for their separate purposes. Brian mentally sectioned and reassembled units until he achieved a particular desired shape of wood. Jack sectioned the plank into units of which he knew the price and so was able to estimate the cost of a plank without knowing the numerical size of its volume.

The concept of place value. In Episodes 15, 16, 17, and 18, the carpenters demonstrated a limited understanding of place value. Their attempts at conversions within the metric system were not consistently correct. Converting from meters to centimeters or millimeters presented fewer difficulties than conversions in the opposite direction.

The concept of proof. The concept of proof is illustrated by the explanations of both Jack and Mr. S in Episode 5. Jack clearly realized that he needed to produce a reasoned argument in order to convince me that he had shown that his square rule was square. When he realized that his first method did not convince me (and, I suspect, noticed the flaw himself) he succeeded in constructing another argument that was convincing, both to me and to himself. He also provided two counterexamples (the "off-square" squares) to further bolster his argument.

Mr. S was also intent on constructing a convincing argument that would impress upon me the fact that his square rule was "true." The process by which his argument developed and improved unfolded around his conjectures and his responses to my awkward questions and to the restrictions I put on which "tools" he was allowed to use.

Summary. From my point of view as a mathematics educator, I found examples of conventional mathematical ideas in many of the explanations, actions, and procedures described and demonstrated by the carpenters in the episodes. By comparing the mathematical ideas across Group One and Group Two episodes, it can be seen that the carpenters favor geometric imagery as a way in which to act upon problematics and that they avoid encounters with algebraic calculations.

In all episodes from Group One (i.e., where the problematics arose spontaneously), geometric methods were chosen and used in action to arrive at a solution. Even Mr. S's design of his conversion method from Imperial to metric units depended on the relative lengths represented by the numbers (see Episode 10). In episodes from Group Two (i.e., where I posed the problems), when I posed a problem that almost seemed to force an algebraic approach (see problem posed in Episodes 16, 17, and 18), the carpenters developed methods that were designed either to minimize the need for calculations (see Episode 18), or to incorporate calculations as secondary to a visual solution (see Episode 17). In Episode 17, Brian finally performed the conventional calculation for volume, after using a previously solved problematic as a model that he applied visually.

The Carpenters' Conceptions of Mathematics and Feelings About Mathematics

"Mathematics" was specifically mentioned on rare occasions in the workshop. However, some very clear attitudes concerning mathematics and the connection of the carpenters' work to mathematics were articulated and displayed in other less obvious ways. Based on an analysis of the third group of episodes, two main themes are elaborated. First, the carpenters did not consider their work to have much to do with "real" mathematics. Jack claimed that they did "a bit of adding and multiplying," which he did not count as genuine mathematics. Second, in those episodes where the problem arose spontaneously in the workshop and the carpenters were able to frame their own problematics, they were more successful, more confident, and more willing to become involved.

Jack was by far the most outspoken of the carpenters and the most ready to articulate his opinion on any matter. Consequently, it is mostly Jack's voice that can be heard in the episodes. He gave a clear indication that as far as he was concerned, no "real" mathematics was used in the workshop (see Episode 4). Furthermore, he portrayed mathematicians as being highly intelligent individuals (if somewhat deficient in other ways!). In Episodes 1, 6, and 12, Jack made attempts to avoid being involved in an activity that he seemed to identify as "mathematics." His motivation appeared to be the fear of being made to feel foolish and inadequate. When Jack identified a problem as being "mathematical," he was sure that there was only one correct answer (see Episodes 5 and 6).

As can be seen from episodes 16, 17, and 18, there were strict boundaries on the sort of mathematical knowledge that the carpenters allowed themselves to own. The "specialized" task of estimating the cost of wood was seen as being Peter's domain and inaccessible to the carpenters (probably partly because of the mathematics involved and partly because the problem concerned money). They portrayed the mathematical knowledge of which they *did* claim ownership as trivial or "simple." For example, Jack stated that they do "just a bit of adding and multiplying, but no *real* maths" (see Episode 4), and Clive remarked, "Here we get only the simple things" (see Episode 13).

A comparison of those episodes that describe encounters focusing on spontaneous problems with those that describe encounters where I deliberately posed a problem highlighted several differences. When they were able to define their own problematics, the carpenters were more likely to draw on their own resources to find a solution and less likely to seek an outside authority. For example, Brian's behavior in Episode 15 conveyed confidence and pride in his solution, but his initial behavior in Episode 16 conveyed uncertainty and a lack of confidence. A further example is provided by considering Jack's behavior in Episode 2, where he conveyed a belief in his own resources, whereas in Episode 18 he appealed to Peter as the external authority. There was also more willingness to engage with a self-framed problematic, and a more confident attitude was evident.

Compared to Jack, Clive and Mr. S were much more circumspect, more accommodating of my questions, and possibly somewhat more confident about their knowledge in general. Mr. S, in particular, seemed to be very comfortable with his knowledge. He strongly valued his skills, experience, and wisdom as a carpenter, and was not intimidated by being asked to engage in novel problems. Although he told me that he was "uneducated" and claimed to know only a "bit of arithmetic," not having studied mathematics at school, he clearly knew that he was able use his experience and his understanding of his environment in a powerful way. His understanding of geometry was a part of his life and had firm links to natural phenomena. He did not defer to authority but preferred to find his own way of understanding things and of creating order and sense in his world (e.g., his use of the horizon as a "natural level" and his creation of a way to "see" metric units).

Summary. While "mathematics" was generally not seen to be something owned and used by the carpenters in their work, they displayed a range of attitudes about being intellectually challenged by questions that drew upon their mathematical experiences. While Mr. S welcomed the intellectual stimulation, Brian and Clive were quietly participative. Jack often felt threatened by my questions. Judging by the question that he raised in Episode 19, I think that with more practice, he would have begun to enjoy taking more risks.

Significant Characteristics of the Carpenters' Mathematizing

The work of Krutetskii (1976) was recognized as being a valuable model for helping to describe the characteristics of the carpenters' mathematizing. Krutetskii declared as one of his research goals "to reflect the basic specific character of mathematics within the framework of the demands it makes on a person's mental activity" (1976, p. 86). In the preliminary analysis that took place during the early stages of data collection for the present study, Krutetskii's "basic characteristics of mathematical thought" (p. 87) were seen to be a good match for the data that I had collected. His categories describing the characteristics of

mathematical thinking became a useful tool for generating questions and for collecting further data.

Krutetskii (1976, p. 87) lists nine "component mathematical abilities that arise from the basic characteristics of mathematical thought," based on the results of a 12-year study concerning the abilities of Russian school children. The following is a summary of that list. The points are numbered K1 through K9 for purposes of reference in the text that follows.

K1. An ability to formalize mathematical material, to isolate form from content, to abstract oneself from the concrete numerical relationships and spatial forms, and to operate with formal structure.

K2. An ability to generalize mathematical objects, to detect what is of chief importance, and to see what is common in what is externally different.

K3. An ability to operate with numerals and other symbols.

K4. An ability for sequential, properly segmented logical reasoning that is related to the need for proof, substantiation, and deductions.

K5. An ability to shorten the reasoning process, to think in curtailed structures.

K6. An ability to reverse a mental process.

K7. Flexibility of thought—an ability to switch from one mental operation to another—and freedom from the binding influence of the commonplace and the hackneyed.

K8. A mathematical memory (i.e., a memory for generalizations, formalized structures, and logical schemes).

K9. An ability for spatial concepts.

Krutetskii worked in the context of schools and so confined his investigations to the mathematics taught in the standard Russian school curriculum. It should be noticed that Krutetskii does not believe that abilities are innate but that they are "developed by living and working" (1976, p. 3). It should also be noted that Krutetskii conducted his research in

a laboratory setting and did not take into account any cultural or social factors that may have influenced the students.

The episodes in Group One and Group Two were examined to identify the characteristics of the mathematizing engaged in by the carpenters. Common themes were searched for within each group of episodes and across the two groups of episodes. Referring to Krutetskii's work helped to provide me with a vocabulary for describing the characteristics of the mathematizing that I discerned in the explanations, actions, and customary practices of the carpenters. Some of the characteristics that I describe are developed inductively from the data. Others were recognized because of the observed similarities to Kruteteskii's findings. These will be referenced to Krutetskii's nine components using the numbers K1 to K9.

Development of concrete problematics and the use of physical actions. One of the most significant characteristics of the carpenters' mathematizing is the way in which they developed concrete problematics in order to generate explanations. Problems that were posed in a deliberately decontextualized form (e.g., "When do you call a line "straight"? and "How can you show that something is straight?" See Episode 20.) evoked similar responses from both Jack and Clive. Both carpenters developed problematics involving pieces of timber and described the physical actions necessary to ensure that the planks were straight.

In Episode 2, when I asked Jack how he knew that a particular point was the center of a box lid, he linked his explanation to a familiar problematic and described the actions he would take to solve that problematic. In Episode 5, my question about how to show that a square rule is square generated the posing of several different problematics, all linked to physical actions.

Ideas are framed by context, shaped by tools. Another significant characteristic of the mathematical ideas developed and used by the carpenters is the way in which their ideas were framed by the context of the workshop and shaped by familiar tools. Their physical

actions and verbal explanations were uniquely those belonging to a carpenter, but the ideas expressed were universal. For instance, the concept of straightness was elaborated using the planing machine, try square, thicknesser, and straightedge as imagined props (see Episode 20). Mr. S used planks, a castor (small swiveled wheel), string, a door frame, and a window frame to construct convincing arguments about squareness and horizontal and vertical lines.

Tools and mathematical formulae. Mr. S appeared to view a tool as something that provided him with a way to replace several indirect actions with one direct action needed to perform a task. The tool could be used as a one-step action, rather like a mathematical formula is one step that can replace a sequence of steps.

In Episode 5, Mr. S drew an analogy between the functions of two different tools. The analogy revealed the actions that are abbreviated by the use of those tools. In his solution showing that a square rule is square, Mr. S elaborated the actions that are replaced by the use of a square rule. The actions replaced by using this tool directly were (a) to find a "level" (horizontal) line—say, a plank—and (b) to find a line—say, another plank—that is "plumb" (perpendicular) with the first plank.

In Episode 8, Mr. S referred to the marking-gauge as being a "shortcut." I assume he meant that using the tool was more direct than taking the two actions that he described. Tools are obviously designed by carpenters to make their tasks easier, to shorten the labor process. This characteristic is parallel to that referred to in K5.

Mathematizing is implicit in actions. Many examples of the mathematical ideas implicit in physical activities are given in the section that describes some of the conventional mathematical concepts underlying the practices of the carpenters. For a detailed discussion, refer to the heading "Conventional mathematical concepts that were embedded in carpentry practice in the workshop."

Abstraction of mathematical ideas from actions. Most often, the mathematical ideas remained implicit in the actions and were not articulated. However, in Episode 2, Jack used an abstraction of his customary physical activity of squaring a table to explain how to find

the center of a boxlid. In his explanation, he used a different representation of the idea that underlies the physical action, thus "isolating form from content" (see K1). He represented the physical action of placing a narrow plank from corner to corner in a frame by referring to the lines drawn on the boxlid, thus using the pencil lines as "symbols" of the narrow plank (see K3).

Generalization to novel situation. Jack generalized the solution of the familiar problem of drawing a square star in order to construct a new solution for drawing a star with an unfamiliar shape (see Episode 1). Although the two stars looked "externally different," he developed a set of rules based on what he saw as being "common" in the shapes of the stars (see K2). Brian generalized the solution of a familiar problematic and used it as a model to solve a novel problem, noticing the elements that were common in the two different situations (see K2 and Episode 16). In Episode 13, after constructing a strategy for drawing a five-pointed and a nine-pointed star, Mr. S made a statement generalizing the methods for drawing star designs with odd and even numbers of points, extrapolating from his particular solutions.

The development of convincing arguments. The logical and convincing arguments that both Jack and Mr. S provided me in their explanations (see Episodes 2 and 5) showed the sequential, logical reasoning that is related to the need in mathematics for proof and substantiation (K4).

Innovative solutions and flexible ideas. Mr. S was able to solve the problematics in which he was engaged in Episode 5 in a number of different ways. All his methods were innovative and certainly "free from the binding influence of the commonplace and the hackneyed" (K7). Jack constructed an alternative solution to his first flawed attempt in Episode 5 when he realized that he did not have an adequate defense for the first solution. Jack and Clive, in Episodes 17 and 18, constructed novel solutions to an unfamiliar problem.

The use of spatial concepts. In order to overcome a lack of knowledge of the conventional mathematical algorithms and poor algebra skills, the carpenters rely on spatial visualization and their own common sense to solve problems (see Episodes 16, 17, and 18). There is a general tendency to use spatial visualization effectively to solve problems (see Episodes 15, 16, 17, and 18).

The carpenters have developed a concept of volume that is rich in its visual and spatial aspects but weak in the numerical calculation of size (see Episodes 16, 17, and 18). Similarly, Jack has a strong visual understanding of the concept of angles, although he is unfamiliar with the conventions of angle measure (see Episode 20). Since the spatial and visual aspects of understanding volume and angles are clearly more important than measuring and attaching numbers to the volumes and angles in the workshop, it is not surprising that the visual aspects are well developed, whereas the numerical aspects are not.

Constructing functional visual units. A significant characteristic of the carpenters' spatial visualizing was their construction of functional visual units when solving problems. Brian's solution (Episode 15) for cutting up a plank into pieces suitable for turning table legs illustrates his visual strategy of sectioning the plank and mentally manipulating and reconstructing units to the size and shape required for the table legs. He produced the *optimal* result of 10 legs, which would not be obtained with formal procedures, such as using the standard algorithm for calculating the volume and then using the division algorithm. His strategy was more than simply a decomposition into blocks, as he did when trying to visualize the calculation of area and volume (Episode 16). In this case, the decomposition was followed by a reconstruction of units, the size and shape of which were chosen to suit a particular function.

Similarly, Jack (Episode 18) mentally decomposed the plank into units that he chose to suit a specific function—that of estimating the total cost of the plank from a unit of which the cost was known.

Comparing versus measuring. The carpenters favored the use of comparison rather than measuring. Comparison employed their senses of touch and sight and often resulted in a more direct and accurate method than would have resulted from measuring. This characteristic will be discussed in more detail in Chapter 6.

Chapter 6

RESULTS AND ANALYSIS: THE WORKSHOP—A CLASSROOM WHERE MATHEMATICS IS TAUGHT AS AN ACTION

Introduction

The aim of this chapter is to address the second research objective of the study—to study the teaching and learning of mathematical ideas that occurred in the context of my apprenticeship in the setting of the workshop. In this chapter, the teaching methods of the carpenters are described from my standpoint as a learner carpenter. New insights into my mathematical knowledge, which resulted from being taught to mathematize like a carpenter, are explored. There is an examination of the social and political issues involved in viewing the workshop as a classroom. Comparisons are drawn between situations in the workshop and similar situations that exist in mathematics classrooms.

In comparison to the style of the previous chapter of results, which gives a detailed description of the analysis process, this chapter is written in the style of an essay that conveys a description of my experiences as a learner. The essay style was chosen for two reasons. First, the data were collected less systematically, relying not on the recording of specific encounters but rather on a historical reconstruction of my experiences in the general educational milieu. Second, the essay developed from examining my field notes in conjunction with my personal diary, and so a narrative form seemed most appropriate.

The Teaching Methods of the Carpenters

I shall be commenting on the teaching methods that the carpenters employed in my own instruction and not in the teaching of other apprentices. Since teaching occurred as a part of ongoing daily tasks and I was fully engaged in such tasks, I was not able to observe any instruction that the other apprentices received. It should also be noted that technically speaking, I was not really an apprentice at all, although Mr. S referred to me as "his apprentice." The title of "apprentice" designates a certain type of training that takes place

over a period of about four years and entails attending a technical college for coursework for a three-month period during each of the four years.

This was the safest experience I have ever had as a novice. Mr. S was a wonderful teacher who had an uncanny knack of arriving at the right moment to render exactly the right kind of assistance. He watched my actions very intently, even if he was not working nearby, and seemed to know when something was going wrong. His method of dealing with this was to proceed with action—sometimes he did not say a word! He would take the tool or sandpaper from me and deftly perform the actions needed to improve what had not gone so well and then would continue until I could see progress being made. Sometimes he would put my hands on the tool and then cover them with his own and continue to work so that I could "feel" how to use the tool correctly. He would point out details like my wrist movements, tension in my forearm, my feet pointing in the wrong direction, and the height at which I was working (sometimes too high). Much of his advice to me was a message to be aware of my whole body and of the rhythm of my movements. He stressed my grip on the handtools often, encouraging me to "try to squeeze so hard that you will break the handle." When he was satisfied that I could proceed alone, and this he would check on by standing back and watching me for a while, he would leave.

He gave me the impression that there was nothing that could go wrong so badly that it could not be rectified somehow. There was no "wrong answer" in the eyes of Mr. S. Perhaps something would not turn out as intended, and modifications might be necessary. At worst one would have to start again from the beginning, but even this would be seen in a positive light because one would have had more practice at the task! His favorite response to my anxious queries when I feared that I had cut something too long or too short or I had scraped too hard was to say, "That's nothing!" He would then take over and demonstrate why I should not be worried.

Jack was also a vigilant teacher who kept a close watch on my progress. He gave me the first lesson on how to use a chisel, showing me first how to hold the chisel,

describing his actions out loud as he demonstrated how to cut away the star shape on my jewelry boxlid. He handed me the hammer and chisel and watched my first attempt critically, stopping me after one knock with the hammer. He warned that I was holding the chisel too tightly and would hurt my hand. I tried again, and again he stopped me almost immediately. I was crouching over the chisel in a tense position—I should relax. He took the tools and demonstrated again. I tried to copy his stance and movements. This time he was satisfied, and I took the hammer and chisel away to my own workbench and continued to hew away the shape of the star to be inlaid on the boxlid.

On another occasion, I was helping Clive assemble some chairs and had to use the electric drill to make some holes into which wooden dowels would be driven to hold the components together. It was important that the holes be drilled at right angles to the seat components. Clive watched the position of my hands and the drill closely and corrected any slant that I inadvertently introduced by pushing the drill into the correct position while I was working. This gave me the opportunity both to *feel* and to *see* the result of the correction.

There was a great deal of support for any artisan or apprentice who needed it. The men watched each other and presented themselves, unasked, to give help or to offer a more convenient tool. Towards the end of my time at the workshop, I was working on a small chest, cutting out dovetail joints with a hand saw. Mr. S stood and watched as I began, correcting the position of my feet and my grip on the saw handle. After he left, Clive came over and took the saw from me, advising me to watch the angle at which he held the saw to ensure that the cut was straight. I tried to copy the angle while he observed me making the next cut. A few minutes later, Jack arrived at my side and took the saw. He cut along the side of one of the dovetails, urging me to guide the saw blade by applying pressure with my forefinger and thumb, as he was doing. Each person had a different piece of advice to offer and demonstrated the efficacy of the practice that he recommended by demonstrating the

action involved. As a learner, I was expected to copy these actions and to understand their importance in the production of work of an excellent quality.

The Carpenters' Tools

It is through teaching a novice the correct use of tools that the carpenters teach the mathematical ideas implicit in their craft. Through the actions of using saws, drills, planing machines, square rules, straightedges, sliding bevels, marking-gauges, sandpaper, and other tools, an implicit understanding of the concepts of "straight," "horizontal," "vertical," "angles," "parallel," "symmetry," "congruence," and "proportion," among others, may develop. For instance, when I asked Jack and Clive about the concepts of "straight" and "square," their discussions were built around how to use the plane, the thicknessing machine, the try square, and the carpenters' square correctly.

Their tools were extremely important to the carpenters with whom I worked. Only the artisans had their own sets of tools, and these were kept sharp, clean, and neatly packed away when not in use. Clive and Jack had lockable wooden trunks that they left in the workshop overnight, while Mr. S brought his tools along daily in a metal bucket. It was my responsibility to watch Mr. S's set of tools for two hours on Fridays when he went to mosque at noon. He allowed me to use anything I needed but frequently warned me not to leave articles lying about unattended. Sometimes I would find that he had put something away that I was still using, simply because I had momentarily moved away from the place where I had been working.

I was given a small "toolbox" after my first few days at the workshop. Mr. S ingeniously made the tool container from two plastic milk bottles with handles on the sides. He cut off the bottoms of the milk bottles and pressed them together, making an airtight case that was useful for storing polishing materials. Inside the container, I stored all the tools necessary for my occupation as a sander and polisher—a large nail, two small nails, and a flat sharpened stick, for cleaning inside carvings, a metal scraper, sandpaper, steel wool, a bunch of horsehair, and the pad of cotton cloth, tightly wrapped in plastic, that I

used for French polishing. About six weeks later, I was given a wooden mallet that Mr. S carved specially to fit my grip. While observing the inlay work I was engaged in, he had noticed that the smallest hammer was too heavy for me to use comfortably, so he designed a more effective tool that was light enough for me to handle with ease.

Mr. S had a particularly rich understanding of his tools that I did not see evidence of among the younger artisans. As discussed in the previous chapter, he appeared to view a tool as something that provided him with a way to replace several indirect actions with the one direct action needed to perform a task.

Learning to Mathematize Like a Carpenter

I noticed with great interest that I was learning new ways to mathematize from the carpenters. There were fertile grounds for learning mathematical ideas in the activities in which I took part as an apprentice. I was fortunate enough to be included in a wide range of activities during the months I spent in the workshop. In many cases, the mathematizing to which I was introduced in the context of the workshop led me to recognize ideas with which I was already familiar and that I had previously learned in an academic situation. The practical experience was fresh and exciting and led me to perceive the world in new ways.

The Development of My Sense of Touch

The first job I was given was to sand an antique walnut standing lamp down to bare wood and then to French polish it. This took several days. On finishing the task, I reflected on what I had learned and noted in my diary:

> I wondered what I had learned from my first job. First, an appreciation of the lovely curves in the design. Watching Mr. S work with the wood and stroke it like he does, I have tried to copy him and now know shape in a slightly different way than I did before. I have always known shape with my eyes, and often only two dimensionally. Now I know it with my palms and fingertips in three dimensions. It is a qualitatively different way of knowing shape. I also know more about choosing something (in this case a small piece of wood to wrap sandpaper around to get to the grooves and curves) which fits snugly into another shape, like a practical version of some engineering aptitude tests. It is essential when sanding something which is carved, to choose a shape which fits evenly into the gaps, otherwise the varnish will be rubbed off unevenly.

For the first time, my sense of touch was trained to check for congruence, to compare lengths, and to explore shapes. Mr. S made frequent attempts to enhance my awareness of my sense of touch. When teaching me how to French polish, he carefully told me how things should feel. For instance, the shellac-soaked inner ball of soft cotton covered with an outer cloth that is used for polishing "should feel like the body of a live pigeon in your hand." In between coats of shellac, he taught me to sandpaper the piece of furniture, "because all the separate grains of wood are standing up—you can feel them with your fingers."

Sanding and polishing tabletops gave me the opportunity to develop a tactile sense of what is both smooth and level. This sense helped me to understand the idea of being "straight" as elaborated by the carpenters. The concepts of being level and smooth were related to the concept of being straight and were used by the carpenters to get a straight edge on a plank as described in Episode 20 in Chapter 5.

When cutting planks to the same size, Mr. S urged me to "feel the edges with your fingers, it is the best way to tell if they are the same length." The sense of touch seemed to me to be intimately related to the ability to visualize and was thus linked to the development of spatial concepts and the ability to design. I had further insight into this idea through Clive's explanation of how to use a small plane to get rid of some "bumps" on a chair component. The part on which I was working was the long piece of wood that forms the side of the chairback and continues down to form one of the back legs. I asked him how I would know when the task was finished satisfactorily. He replied, rubbing the wood reflectively, "It must be a smooth flowing shape, it must flow evenly, like this (moving his hand down the length of the wood)...it must be one continuous line."

The furniture in the workshop was always being touched and stroked. On several occasions when I asked one of the carpenters whether or not he liked a piece of furniture, he would comment on its tactile qualities as well as other issues of design and construction. It occurred to me that my formal academic mathematics education could have been greatly

enriched by developing my sense of touch, not only in the elementary stages where the value of concrete manipulatives is now well recognized, but, for instance, in the study of calculus, to "feel" a smooth curve, or a point of inflection. Exploring the shape of the hull of a yacht with my fingertips along with an explanation of how calculus is used in the design would have added considerably to my understanding of calculus.

Learning How to "Use My Eye"

Learning how to "use my eye" is an extremely valuable lesson for a novice carpenter. "Having a good eye" is synonymous with being a good carpenter, and being told that one "has an eye" for a task like designing, carving, or lathe work is a great compliment. Although no direct allusions were made, it was clear that the carpenters tried to train my eye while they taught me the basics of woodworking. In the mathematics classroom, I had been drilled in the more conventional and "accurate" practices of measuring lengths of lines with a ruler and sizes of angles with a protractor on the two-dimensional page. In the workshop, my ability to make judgements by sight was developed by being encouraged to compare lengths or to check whether a line was straight or a surface was horizontal "with my eye." It was in this way that the carpenters became adept at visual approximations. The concepts of being straight and level were given concrete meaning in three dimensions and were linked to actions.

On one occasion, I was helping Mr. S reconstruct a large antique table top that had broken apart into four separate planks. The planks, which were slightly warped, did not have parallel edges and did not fit together snugly any longer. The curved lines and cracks had to be preserved while the planks were being fitted together, since creating straight line joins would not have preserved the aged appearance of the table. A kind of slow jigsaw puzzle was created as Mr. S decided where to alter the shape of the planks using a small hand plane. Sometimes he chose to smooth away a bump that was situated far away from a troublesome gap. To my surprise, the removal of a seemingly unrelated bump caused a snug fit where the gap once was! I could not discern how he did it, nor could he tell me

what to look for. The enigmatic "I see it with my eye" had to suffice. He frequently asked me for "advice" on where to plane. Sometimes he would follow my suggestion and would be able to demonstrate practically that it did not work. He would then tell me his choice, perform the actions, and demonstrate that his strategy did help. In this way, he tried to train my eye.

On another occasion, Mr. S asked for my assistance when he had to drill into a table leg to make a housing for a very thick dowel that he intended using to replace the bottom of the leg that had broken off. He lay the table on its side and asked me to help him by guiding the drilling so that the dowel hole would be straight. He instructed me to squat down so that my eye was at the same level as the drill bit. My task was to "use my eye" to line up the drill bit parallel to the far opposite leg and then alert him if the drill was dropped down or pulled up in such a way that it was no longer in line with the opposite leg. When he had finished the drilling, he invited me to "sight" the leg from the bottom, to check that the hole was located in the center of the round end face. He then fixed on the bottom part of the leg and invited me to "sight" it both from the bottom and the side again, to check that it had been fixed on straight. This skill is obviously very important to him and he spent a lot of time making sure that I was "eyeing" everything correctly.

Learning to use one's eye in these ways develops the ability to visualize, to "see in the mind's eye," and is an important part of designing to ensure balance and harmony in a finished piece of furniture. According to Peter, "There is a skill or an eye in being able to visualize what you want to build that makes you better than some" as a designer. On several occasions, Mr. S took me round the workshop and criticized some furniture as a learning exercise for my eye. He pointed out a carver with big curved arms and said that "to his eye" the arms were too heavy for the thin turned legs. The chair looked top-heavy. He pointed out table legs that were too light and narrow or too heavy for the size and thickness used for the tabletop. He paid at least as much, if not more, attention to the aesthetic quality of design for furniture as he did to practical considerations.

Learning When to Measure and When to Compare

As described previously, after six weeks of sanding and polishing, I moved into a new phase of learning and began to help in construction tasks using the electric saws and other equipment. It was at this time that I began to notice that I had a tendency to use a measuring tape more frequently than Mr. S considered desirable. Mr. S made a clear distinction between the two operations of "comparing" and "measuring." By "comparing," I mean, for example, the act of checking whether two geometric or physical lengths are congruent, or one is shorter than the other, or one is twice the length of the other. The two lengths could be checked directly or could be compared to a third length. Checking can done by the eye and by feeling with the fingertips, and a high degree of accuracy can be attained. By "measuring," I mean assigning a number to a geometric or physical length. The number assigned to indicate the length will depend on the units used.

My first impulse was to reach for a tape measure whenever I had to cut a piece of wood, compare lengths, or copy a design. In most cases, however, Mr. S did not use a tape measure or a ruler but preferred to use his eye or to mark off the length required with his hands or fingers. Sometimes he used a stick and marked off a length using a pencil and then used this as a comparison tool. In many cases this practice was much more convenient than measuring with a tape or ruler. He took great care to tell me when I did not need to use a tape measure and encouraged me to use his methods.

Comparison often turned out to be the most direct and accurate way of completing a task. Frequently, the assignment of a number would be an unneccessary step. For example, when squaring the aprons that support a tabletop, it was not necessary to know the lengths of the diagonals. It was enough to know whether they were the same or different. If they were different, the very stick being used to compare the diagonals would be cut and used as a rigid bar to push the aprons into the correct "square" position. Thus measuring to the nearest fraction of an inch or millimeter with a tape would simply be extra work, as well as less accurate.

As another example, when cutting planks of wood to match the size of one that had already been cut, a comparison often decided where the new cuts should be. Sometimes the plank that was already the correct size would be laid on top of a longer one and a pencil mark made. A square rule would be used to "pull" the line across where the new saw cut should be made. If there were many new planks to be cut, a thick chunk of wood would be nailed onto the wooden table of the table saw as a stopper at the 'eft-most end of the original plank, while the right-most end was against the saw blade. The new planks would pushed up against the stopper as a frame of reference and cut with no measurements taken. Fingertips would be used to check for congruence.

On one particular occasion, described in Episode 7 in Chapter 5, measuring actually got in the way of the task at hand, and I realized with great clarity what Mr. S had been trying to teach me about the difference between measuring and comparing. Mr S's solution illustrates his understanding of proportional size and his appreciation of the geometry of the design and of the importance of the internal relationships of the design. His solution avoids the use of measurements and relies on a more direct method of comparison. The elegance of his solution lies in its simplicity and conciseness and its reliance on the geometric interrelationships of the piece of furniture. Brian and I fragmented the design and the tasks, while Mr. S took a holistic approach.

When Mr. S did decide to measure, he knew that the unit was his own choice. Sometimes he selected Imperial units when restoring old English or colonial furniture, because that is what the original craftsman had used. On other occasions he used metric units. The metric system was obviously less familiar to him, but he had worked out his own conversion system from the Imperial system in order to make sense out of it.

Making the distinction between comparing and measuring drew attention to the idea of choosing a unit. I recognized from my academic mathematics training that these basic ideas underlie the concepts of vector spaces. Abstract vector spaces do not have units or coordinates assigned, and two lengths are added by juxtaposition (i.e., like putting two

planks next to each other to figure out the length of a third plank). Scalar multiplication is done by magnifying (e.g., the inlay design was magnified by one half). Choosing a unit (e.g., Imperial or metric) is the same as picking a basis for a vector space. In the vector space, the coordinates with respect to the basis are the "measurements." These are things that I have known for some time, but it was not until I was encouraged to reflect upon my experience in the workshop that I was able to attach a practical meaning to my knowledge.

Routine Activities in the Workshop That Did Not Encourage Mathematizing

I do not wish to claim that all the activities in the workshop required the carpenters to mathematize or even gave them opportunities to do so. Much of the production work was streamlined and performed in a rote, mechanical fashion, where speed was more important than reflecting on the task at hand. The carpenters' problematic then was to get the components cut as quickly and with as close a match as possible to the templates that Peter made.

The men were obedient to a set pattern of action. The pattern had developed over time, and while I am sure that the workers contributed to its development, there seemed to be a set rhythm and style for some jobs that discouraged deviation. For instance, once Peter had designed a chair, the artisans would cut out templates of the component parts and make and test one prototype. If it was decided that production should proceed, the templates were used to cut the components in bulk. Cutting was done by one or two men (usually the apprentices), using a manually operated electric saw. The chairs were joined with mortise-and-tenon joints (see Figure 51).

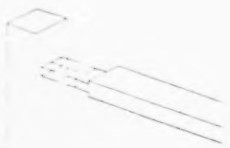

Figure 51. Mortise-and-tenon joint.

One of the apprentices would make the mortise cuts required on certain components using the electric overhead mortise cutter. This was a mechanical task where the depth of the cut could be set on the machine and the length of the cut made in accordance with two pencil marks. Later, a third person would make the tenon cuts on the components, and it was then mainly that third person's responsibility to assemble the chairs.

The fragmentation of these tasks may have resulted in some advantage for production but certainly led to other difficulties, since the first group did not have to take responsibility for the assembly of the chairs. They did not participate in solving the problem of how the chairs should fit together. Thus their problematic did not extend beyond the cutting of the components. The person making the tenon cuts often found it difficult to decide what his fixed point of reference should be before starting to cut and so made repeated errors. When the chairs were finally put together, a lot of time-consuming and labor-intensive handwork had to be done to get the chairs to meet the standard required. All in all, the cutting of chair components seemed to be done rotely and without much cognitive engagement on the part of the carpenters, rather like a student might apply algorithms and definitions rotely when solving a routine algebra problem.

The Workshop as a Classroom

In exploring the analogy of the workshop as a classroom, the social and political issues come to the fore. I recognized that many of the barriers to learning that are present in classrooms were also evident in the workshop. Unlike the structured teaching-learning situations that one expects in a classroom, the situation in the workshop seemed at first glance to be much more relaxed and informal. I had expected to find the knowledge of the carpenters accessible to everyone in the workshop.

However, a hierarchy existed that resulted in the poor distribution of learning opportunities across members of the workshop group. A situation similar to the labeling of students as "more able" and "less able" that often takes place in mathematics classrooms was found in the workshop. The laborers were the "silent members" (Volmink, personal

communication, September 1989). In the case of this particular workshop, the restrictions placed on the behavior of the individual and his access to knowledge were exacerbated because of the pervasive and pernicious atmosphere of apartheid.

External authority for certain ideas was seen to be vested in Peter, or in a calculator, or in a better-informed friend. In some circumstances there was evidence of a search for the "right answer." As in classrooms, products were offered for the scrutiny of the group and of the authority figure and found to be lacking. Criticism was not always constructive.

Traditionally, carpentry has been the preserve of "coloured" men in South Africa. Women are never hired, and black men in a carpentry workshop are almost always hired to sand, clean, and finish furniture and to sweep the floor. They are generally seen as a laboring and unskilled group, unlikely to ever have the desire, need, or ability to learn woodworking skills. An example of someone who fell into this group was Sidney, who had been involved with restoration and construction of both houses and furniture for at least 20 years, 18 of these years in affiliation with Peter. Sidney was an expert at identifying wood and at working on percentage reproduction (when new parts are added and are "aged" to match the original pieces), but he had never made any article of furniture of his own. A major part of his working life took place during the heyday of "grand apartheid," when jobs for blacks were even more severely restricted than they are today, and it would have been unthinkable for him to have tried his hand at woodworking. At the time of this research, he was an old man and was unlikely to start anything new during his last years before retirement.

Peter had tried in his workshop to encourage everyone to work on his own woodworking projects at lunchtimes. The men were allowed to use wood offcuts to make whatever they could and were encouraged to use the machinery at lunchtime. Many of the younger laborers took advantage of this, and I saw several small coffee tables, hinged boxes, a breadbox, and a cabinet being built. However, there was no organized plan to familiarize the laborers with the basics of woodworking, and they received very little

encouragement and only grudging assistance from most of the artisans. Work done by the laborers was regarded with some derision by the artisans, and their efforts received a lot of unconstructive criticism.

There was a sense that everybody knew his place and that there was little chance to change that. Each person knew what the expectation was of his performance by the group as a whole as well as by Peter. All the laborers were black men. The laborers were not expected to do woodwork construction but were involved in sanding, polishing, spraying, and finishing furniture. There was an unspoken delegation and demarcation of tasks that resulted in a poor distribution of learning opportunities for the black workers.

The artisans and apprentices (with one exception) were white and "coloured" men. The one exception to this was Leonard, a young black man who was Sidney's nephew. As described in Chapter 3, it was clear that Leonard achieved his goal of becoming an apprentice through a combination of assertiveness and hard work, backed up by a set of unquestionably excellent academic results. Peter was obviously concerned with the welfare and progress of all his workers and clearly acted in Leonard's best interests. However, Leonard was not sought out and offered an opportunity but seized that responsibility himself.

Compare this to the opportunity given to Brian, who had demonstrated no ambition or interest in his job, had habitually arrived at work late, and had an abysmal school record. Peter said that he had been suspicious of Brian at first, since he was a young white man wanting to join a trade that was traditionally reserved for "coloured" people. As a white person, Brian had so many other opportunities open to him. Peter obviously expected more from Brian and so gave him a golden opportunity to achieve more. As it turned out, Brian did have a particular talent with the lathe (that may have been latent in others not given similar opportunities) and was soon promoted to artisan without any formal technical training.

This story was not intended as a criticism of Peter, who had created a particularly harmonious workplace and who strove to give opportunities to all the workmen where none had existed in the past. The point is that South African society has different expectations of its people, based on race, and this to a large extent determines access to knowledge and opportunity, even in an informal educational setting like the carpentry workshop.

As a white woman, I was accorded a privileged status and was allowed an opportunity to study the craft and culture of carpentry. However, the position of bona fide worker was by no means automatically attained. I had to work hard to get beyond the stage of being treated politely as a visitor, to earn the confidence of the carpenters, and to be included in general workshop activities. Carpentry is considered rough and heavy work, suitable only for a man. My gender clearly raised some doubts about the sort of work that I would be capable of when I first started at the workshop. As time went by, I demonstrated not only that my hands could get dirty but also that I was able to do fairly heavy manual labor, and I was accepted as a temporary member of the team. However, whenever any work came up that did not require heavy manual labor and therefore looked vaguely "feminine," it was automatically assumed that I would be the best person to do the task. For instance, I spent a week lining drawers and cutlery trays with green baize, replaced the green baize on an antique card table, and was asked to do the fine gilt painting on some chairs. As in Leonard's case, it took assertiveness and detemination to ensure that I did not remain strictly within the narrow confines of society's expectation of tasks that are suitable for a woman.

In the workshop setting, where quality, profit, and productivity are the watchwords, anyone who can enhance these in any way will be rewarded. Ostensibly, all the experienced craftsmen were willing to share their skilled knowledge. Group discussions of woodworking problems were fairly common, and in general the carpenters preferred to try to solve problems together before taking a question to Peter. The instruction of apprentices, whether as a formal or informal requirement, is part of the job of a master craftsperson.

However, there were complex social obstacles at all levels. I found the artisans hesitant about any tasks that they designated as being in a "specialist" category and "above" them, even when they were capable completing the task. At the same time, these artisans were often unhelpful and critical of laborers who needed assistance with their private woodworking tasks. There was a prevalent perception that certain tasks belonged to certain groups of people, and there was resistance to any changes in this fixed structure, even though everyone would benefit in the long run from such changes.

The issues of power and access to knowledge are thorny topics, and no quick and easy solution can be expected. This is as true in the academic world as it is in the carpentry workshop. These observations have tempered my optimism about the "unfreezing" of mathematics in traditional crafts, or the introduction of curricula that focus on "people's mathematics" leading to an automatic democratization of Western mathematics.

Summary

The account of my learning experiences in the workshop highlights three main themes. First, I commented on the teaching methods of the carpenters, who were operating within the apprenticeship model of instruction. The apprenticeship model will be further discussed in Chapter 7. Second, working as an apprentice provided me with the practical experience of learning mathematics through actions. The carpenters coached me in the use of woodworking tools and in the use of traditional practices such as "using my eye." In the process of learning these actions, I also learned new mathematical ideas.

The third theme concerns the wider educational milieu in the workshop. As in classrooms, there existed hidden barriers to learning that resulted in a poor distribution of opportunities within the group. Some members were relegated to fragmented and repetitive tasks that robbed them of opportunities to mathematize. They went through the motions of the activities but were not provided with the exposure that encouraged fruitful outcomes. Sometimes it was a subtle nuance that took away the opportunity, sometimes a more deliberate step.

Chapter 7

DISCUSSION

Introduction

This chapter is organized into five sections that discuss the various practical and theoretical implications of the present study. I will begin by discussing some epistemological implications for mathematics. The second section focuses on the educational implications of the study. The third section examines some of the methodological issues related to the study, which may be of benefit to those interested in exploring ethnomathematics. In the fourth section, I present some suggestions for further research. The chapter closes with a section entitled "Epilogue."

Epistemological Implications: Knowing in Action

A question that I have asked myself continuously throughout the present research, and one that I think should be asked more often by mathematics educators is, "What is mathematics?" While no definitive answers were expected to emerge from the present research, the account of the carpenters' activities may perhaps cause a few dedicated formalists to ponder the above question and the question of what it means to do mathematics.

I have argued that the carpenters "do mathematics" in the workshop, according to the criteria that I set up in Chapter 1. Their mathematics has some unique characteristics that are different from the mathematics that we are accustomed to seeing in textbooks and that show how the culture of the workshop placed an indelible stamp on the mathematical ideas that were developed there.

First, *action is vital in their mathematics*. There is tacit mathematical knowledge in their physical actions. Physical demonstrations formed part of active explanations, and there was movement and an involvement in the environment. For example, Mr. S's hand actions when demonstrating how the new escutcheon would be flipped over to match the existing one showed a tacit knowledge of reflective symmetry, and his actions of turning the

separate petals in the design on the inlaid table showed a tacit understanding of rotational symmetry. Jack's actions when squaring a box carried implicit knowledge that the diagonals of a rectangle are equal.

Second, *reflection on action often leads to the articulation of tacit knowledge*. For example, Jack's reflections on the actions he took when squaring a table led to his explanation of how to find the center of a rectangular boxlid, while Clive's reflections on the actions he took when planing the edge of a plank and using his eye to check that the edge is straight led to his discussion on the concept of straight lines.

The results of the present research suggest that physical actions, reflection, and knowledge of mathematics are linked in the practice of carpentry. Other researchers have linked action, reflection, and knowing, in various practical settings. In particular, the present research with carpenters can be seen as an extension of the work of Schon (1983, 1987), who argues for a new epistemology of practice, claiming that successful professional practitioners like architects and psychologists display a competence that does not depend upon being able to describe what they know ("tacit knowing-in-action") and who engage in "reflection-in-action" at work. The present study differs from the work of Schon in its focus on the *physical* actions and reflections of manual workers at work, while Schon examined the less obviously physical actions of, for example, architects at work.

The work of Gerdes (1986), who argues that mathematics is "frozen" in traditional crafts and cultural artifacts, is also extended by the present study. One of the shortcomings of the work of Gerdes is that few guidelines are given as to how one practically goes about "unfreezing" the mathematical ideas to which he refers. The examination, in accordance with the theoretical framework I sketched out, of the actions and the reflections of people who are physically engaged in problematics related to woodworking practices that have been "handed down" (note the physical action implied) from father to son, represents a practical suggestion for Gerdes's "unfreezing" proposition.

I claim that the evidence provided in the present study suggests that mathematical knowledge can be implicit in physical actions. Acknowledgement of this claim requires a shift in the accepted epistemology of mathematics.

The third unique characteristic of the carpenters' mathematics is that their explanations, discussions, and problem-solving activities were deliberately linked to concrete, contextual problematics that were used to provide physical props, mental models, and analogies. Questions that I posed in decontextualized forms were re-interpreted by the carpenters, who replaced the decontextualized forms with "real-life" problematics to solve instead. For example, when I asked Jack how to find the center of a rectangular plank, he derived his explanation from a discussion on how he would "square" a table, and when I asked Mr. S how to check which of two different square rules were true, he developed the problematic of designing a method to make a square rule. *I claim that the carpenters' contextualization of problematics does not imply that their mathematics remained at the concrete level and was never abstract.* Examples show that they were able to generalize solutions to new situations, and to discuss abstract ideas, but that *they used different symbols from those conventionally accepted as mathematical symbols.* Contextualizing problematics for explanations and teaching resulted in a rich and vivid portrayal of concepts from my point of view as a learner.

The fourth unique characteristic is that a significant role was played by the carpenters' tools in the shaping of their mathematical ideas. Many mathematical ideas were expressed through the medium of a woodworking tool. *I claim that the carpenters' tools were used as important symbols in their mathematics.* As mathematical symbols are manipulated to illustrate explanations in formal mathematics, so the carpentry tools were physically manipulated to communicate explanations in the workshop. Some of the tools "stood for" actions: for example, Mr. S gave a demonstration of the actions abbreviated by the carpenters' square.

As a collection, these unique characteristics describe a mathematics that does not fit the conventional mold. *I argue that the conventional epistemological view of mathematics should be broadened to admit the activities of the carpenters as an authentic experience of making and using mathematics in the context of the workshop.*

An example of how a broader view was taken of what should be allowed to count as valid mathematics may be found in the work of Confrey (1990). As a result of the examination of student learning, multiplication schemes that did not fit the conventional model of repeated addition were noticed. Instead of the schemes being dismissed as deviant and in need of remedy, they were explored for their strengths. As a result, Confrey (1990) put forward an alternative concept of multiplication described as "splitting." The "splitting" model is claimed to be a more appropriate explanatory model for exponential and logarithmic functions and is independent from, but complementary to, the customary model of multiplication as repeated addition. Broadening our view of mathematics will thus require a tolerance of ideas that are different from the norm.

Several examples of the carpenters' mathematizing (e.g., Mr. S's method of situating dovetails evenly along the edge of a plank, and his method of placing holes along the frame of a chair seat for the threading of rattan (and for the threading of riempie) were found to be consistent with the "splitting" model of multiplication. I believe that the arguments for challenging the accepted conceptions of mathematics will be strengthened if links continue to be found between students' mathematics and mathematics as it is practiced in everyday settings.

As a further challenge to the epistemology of mathematics, I have suggested using the term "mathematizing." By using the verb "mathematizing" (attributed to Weyl, see Footnote 3) in this work, I have tried to emphasize the *experience* and *process* of mathematics. Mathematics is something dynamic, always being added to or subtracted from, something in which people can participate. The noun "mathematics" usually implies something unchanging, set, and static; it is not an experience; it does not invite

participation; it is a cut-and-dried body of knowledge that can be learned about but not experienced in action.

Creating a new word in the form of a verb from the conventional form of a noun has been done in other areas as well, such as in the discipline of linguistics, where "languaging" has been created from "language" (Maturana & Varela, 1987), and in the discipline of law, where "lawyering" has been created from "law" (Schon, 1987). In both cases, the invention of the new word reveals a desire to go beyond the strict limits of a highly structured and rule-bound, abstract domain. The fact that the new words are verbs formed from nouns signifies the desire for action and a rejection of the sense of an "accomplished fact" conveyed by the noun. Becker (1989), in discussing language, makes the following statement:

> Linguists and others have tended to see language as an abstract structure and/or system; a set of general rules of constraints operating on or within a lexicon, i.e. a grammar and a dictionary. In short, as an "accomplished fact" . . . grammars and dictionaries are things made by linguists and have little to tell us about what happens when we "language." The shift from noun to verb, from *language* as an abstract thing of some sort to *languaging* as an act, . . . make(s) a clear distinction between *language*, a description produced by an observer, and *languaging*, an act experienced *in statu nascendi*. (p. 6)

It is with intentions similar to those implied by the writings of Becker and of Schon that I have suggested the shift from the conventional noun "mathematics" to the verb "mathematizing" to signify a shift from mathematics as an abstract "accomplished fact" to mathematics as an action, something that people do, something not completed, but still in process. *On the basis of the results of the present study, I argue for an epistemology of mathematics that is able to accommodate the word "mathematizing."*

Educational Implications

Three main educational implications of the present study are discussed. First, I explore the meaning of understanding a concept and reflect on the ways in which competence in a concept is often judged in classrooms. Second, I comment on the possible use of the apprenticeship model as a viable model of instruction for mathematics education

in schools, and last, I explore some of my emerging conceptions about the significance of considering mathematics education as praxis.

The Meaning of Understanding a Concept

A significant educational question that my experience with the carpenters raised in my mind is, "What does it mean to understand a concept?" We often describe a concept by invoking the standard algorithm for its calculation, so "volume is length times width times height" seems an adequate explanation for the concept. It is easy to confuse the two separate issues of the meaning of the concept of volume and of knowing how to calculate volume, and to thus accept that knowledge of the algorithm is sufficient to prove an understanding of the concept.

The present study provides evidence that understanding the concept of volume means much more than knowing the conventional algorithm used for calculating

$$V = L \times W \times H,$$

where V represents volume, L represents length, W represents width, and H represents height. Peter was mistaken when he told me that the carpenters did not understand the concept of volume. The fact that the carpenters were unfamiliar with the standard algorithm for calculating volume simply implied their lack of expertise with the accepted conventions surrounding the concept and not a lack of understanding of the concept of volume itself.

By school standards, the carpenters would not have been considered to have an understanding of volume, since the algorithm was unfamiliar to them. Yet they demonstrated a rich and robust understanding of the concept, based on spatial visualization, and not on numbers. They were able to use their concept of volume in verbal and in physical demonstrations to generate explanations and to solve problems relevant to their purposes as carpenters.

I wonder how the problem that Brian solved, concerning how many legs he would be able to make from a certain plank of wood, would have been solved by those people for whom the algorithm for calculating volume leaps to the fore at the mention of the word

"volume." An attempt could be made to solve the problem by working out the volume of the block required for each leg and dividing this into the volume of the whole plank. The answer (11.15 legs) produces practical problems. First, this method does not provide a scheme for cutting the planks, and second, although the volume of the offcuts may be enough for an eleventh leg (Brian's scheme allowed him to cut ten legs), it would be undesirable if not impossible to make a leg from scraps. I am not disclaiming the importance of being able to calculate volume numerically. Rather, *I am claiming that the usual demonstration of competence in understanding the concept of volume (i.e., using the algorithm to calculate) looks rather thin in comparison to the knowledge displayed by the carpenters.*

As pointed out by Schoenfeld (1988, p. 2), mathematics educators need to use "broader definitions of mathematical understanding than 'mastering symbol manipulation procedures'," since there are "dangers to the narrow assessments of competency that are currently employed." In many classrooms, competence can be adequately displayed simply by imitating the procedures of the teacher and by repeating symbolic manipulations in learned algorithms. Students are often not encouraged to build multiple representations of concepts. The example given by Saxe (1988) (see Chapter 2), who explored the problem solving of carpentry apprentices and master carpenters, further illustrates my point. The carpentry apprentices, who had all learned the algorithm to calculate volume in formal mathematics lessons, were unable to solve the problem "How much wood needed for five beds?" and offered absurd answers. The master carpenters, who had had considerably less formal schooling and had learned through apprenticeships and experience, were all able to solve the problem with ease and without recourse to the algorithm. This suggests that mathematics teachers ought to provide contexts for students that demand more strength and versatility from concepts than the symbolic manipulation that suffices in most classrooms at present. *The results of the present research underscore the recommendations of the*

constructivist literature that students would benefit from being encouraged to construct and use multiple representations of their concepts.

Apprenticeship as a Model for Instruction

Many students find it difficult to be courageous about learning something new in mathematics classrooms. Often, their objective is to get the right answer as quickly as possible and to move on. The product (i.e., the right answer) in this case is more important than the process. *The apprenticeship system (according to my personal experience) provides a context in which it is safe to be daring about learning new things. There are no right or wrong answers, since the process is part of the solution.* The aim is for faultless performance, but this is not expected to occur without mistakes along the way, without efforts that fall short of the desired goal. Practice is valuable; the building of experience, with success and failure along the way, is the way in which learning occurs. Collaboration is important: for encouragement, for joint problem solving, for sharing new ideas, for teaching novices, for reaching new levels of development with the help of a more expert colleague. Through coaching, the master teacher provides a model of a competent practitioner for apprentices. Apprentices are encouraged to appreciate the products as well as the process of creation.

Apprenticeship carries with it the idea of craftsmanship, since it is traditionally the crafts (thought of as nonacademic) that are taught via apprenticeships. (Note, of course, that medical doctors serve a kind of apprenticeship, but this has been re-labeled as "internship"). Jack made an interesting comment about craftmanship in carpentry that made me reflect more deeply about the effects of fragmenting tasks, formalizing certain steps, and encouraging repetition. He said:

> Here we are working with the solid woods, but there is a complete difference between us and other factories. There they work with laminated chipboard. A complete different story. When you work in any other factory, which I have seen and I know, if a man screws in screws today, then he screws screws for the rest of his life. If he does joining up of drawers, then he makes drawers for the rest of his life. Here you work from the floor, right to the finish of that particular item, and that is what I call a cabinetmaker...a craftsman.

Jack implied that a genuine craftsman performs a task that has integrity and that is neither fragmented nor completely rule-bound. The craftsman would design a plan of action. The task may have different parts, but each part would be integrated into the whole. Above all, the process would be satisfying, meaningful, and engaging to the carpenter.

It is instructive to look at these ideas in the workshop setting, far removed from a conventional classroom, and to reflect on what we, as mathematics educators, can learn from Jack's description of what transpires in different workshops. When fragmentation, formalization, and repetition deprive carpenters of the opportunities to work as craftsmen, are they simultaneously deprived of the opportunities to mathematize? Fragmentation, formalization, and repetition are the hallmarks of mathematics instruction in many classrooms. *What would be analagous in the classroom to the students working as "craftspeople" in mathematics, to working "from the floor, right to the finish"?* The problems that students currently are given to solve in class would probably be described as "laminated chipboard." What would count as "solid wood" in the realm of mathematics problems?

Mathematics Education as Praxis

The experience of conducting mathematics education research in a practical work setting created an appropriate context in which to consider the meaning of praxis. Lefebre (1968) defines praxis as comprising two main ideas. First is the notion that praxis involves action as opposed to philosophical speculation, and second is the view that the fundamental characteristic of society is material production to meet basic needs. He suggests that man primarily acts on the natural world and only secondarily thinks about it.

In an educational context, Fasheh (1988, p. 4) has argued that "education perceived as praxis is the opposite of hegemonic education." He describes praxis as the combination of the social, cultural, and material conditions, in constant interplay with reflection and action (Fasheh, 1988). Fasheh (1988) continues:

> Hegemony is understood here as a form of domination. . . . Hegemony is always linked to an ideology which reflects the manners and interests of the invaders and

> their culture. . . . Crucial to the hegemonic relationship is the invaded's belief that the lifestyles and values of the hegemonic group are inherently, naturally and objectively superior. Hegemony is successful when the invader's ideology is . . . assumed to be universal and superior; when, like the mathematics I valued, it is believed to transcend class, gender, culture, and national boundaries. (p. 3).

Fasheh's reference to the mathematics he valued forms part of his argument that hegemonic mathematics education "renders marginal, deems inferior and makes invisible" (1988, p. 3) mathematics like that practiced by his illiterate, seamstress mother (i.e., mathematics that does not fit the dominant view of mathematics).

The perspective of mathematics education as praxis is consistent with the shift in the epistemology of mathematics that is argued for in the previous section. Mathematics education needs to extend the invitation to students to participate in the experience of mathematizing. In contrast to the notorious reputation that mathematics has earned for being an obscure, difficult, and irrelevant subject to be avoided at school, mathematics educators need to convince people that the experience of mathematizing can be a vibrant, exciting, and empowering facet of their lives.

If mathematics educators continue to ignore the mathematics that is generated in everyday settings and continue to work strictly within the narrow framework of formal mathematics, mathematics will continue to be uninviting and disempowering to the majority of students. *We need to bring nonconventional mathematics into classrooms, to value and to build on the mathematical ideas that students already have through their experiences in their homes and in their communities.*

Methodological Implications

The present study outlined a methodological procedure for conducting practical ethnomathematics research. For mathematics education research, particularly for exploring questions about ethnomathematics, the approach of ethnography provides a useful methodological framework. An ethnographic approach, with an extended period of fieldwork, allowed me the time to explore the ideas of the carpenters as they were engaged in their daily tasks. Because of the length of time I spent at the workshop and because I

participated as an apprentice and bona fide worker, I was able to observe spontaneous problem solving in context. As as participant, I was afforded the opportunity of developing a thorough understanding of the context of the workshop, which assisted in the interpretation of the mathematizing I observed and in which I took part.

The master-teacher–apprentice relationship that was nurtured between Mr. S and me proved to be a fruitful source of data. Mr. S felt responsible for my education as a learner carpenter and strove to provide me with a wide range of woodworking experiences during the time I spent at the workshop. As his pupil, I was in a privileged position in that he taught me certain things that he kept secret from the other carpenters. He guarded some of his knowledge closely and instructed me clearly on what could and could not be mentioned in front of the other workers. His acceptance of me as his pupil contributed significantly to my credibility with the other carpenters, in view of his senior position. Consequently, my ability to collect data was enhanced in a way that may not have occurred through other data collection procedures.

The negative side of this intense teacher-apprentice relationship was that it was sometimes difficult for me to work with the other carpenters at a time when they were doing something interesting. I had a responsibility towards Mr S as my special teacher and respected his allocation of tasks to me.

My experiences with the present research permit me to sound some warnings, however. Ethnographic research is particularly demanding and time-consuming, since it requires extended periods of fieldwork. Locating a suitable site for conducting fieldwork is no simple task, since there may exist unanticipated barriers that hinder entrance to the field. Finding an accommodating "gatekeeper" and a suitable site are two of the first hurdles to be cleared; there are several more hurdles that make ethnographic research a risky and challenging methodological choice for a dissertation. The question of whether or not one will become a participant cannot be judged ahead of time. If one is not designated as a participant by the group being studied, the main method of ethnographic data collection will

be observation. Although observation data may suffice for some research designs, the present study would not have succeeded had I not been accepted as a participant by the carpenters.

Limitations of the Study and Suggestions for Future Research

The results of this study have generated a number of questions worth further research. The perennial questions about what mathematics is and what it means to do mathematics, remain, of course, unanswered. Asking these questions in nonacademic settings, however, will produce an intriguing set of answers. It may be interesting to replicate this study in a different context and to compare the characteristics of mathematizing across contexts.

Further research on the finding that physical action, reflection, and mathematical knowledge are interlinked should be conducted. Of particular interest here would be the development of curricular material that exploits the actions associated with mathematical concepts. For example, students could be taught geometry through involvement in practical projects.

In reflecting on the significance of education as praxis, I came to realize the significance also of research as praxis and to consider the limitations of the present study. Patti Lather (1986) has defined "research as praxis" as the act of researchers involving the people whom they research in a democratized process of inquiry, characterized by negotiation, reciprocity, and empowerment.

After five months of intensive study at the workshop, as trust developed between some of the carpenters and me, opportunities began to arise where it seemed that I could explore the carpenters' mathematics with them in a way that would be empowering to them. In particular, Jack had become curious about some ideas that he had and had overcome his initial desire to hide what he was unsure of from me. However, I resisted every opportunity that arose because I felt that I had to remain faithful to my methodological framework, and I did not want to compromise my position as an apprentice. My role as an apprentice was

central to the argument I had put forward for being able to describe the mathematizing I was learning, and I wanted to remain the learner and not take on any part of a role as teacher. Thus my research could not be described as praxis. My own actions and reflections in the context of the workshop and the nature of the interactions that I could have with the carpenters were severely curtailed by my adherence to my chosen methodological stance. Similarly, in the case of the laborers, I noted the inequities of their access to learning in the workshop but could not use my research to try to raise their awareness of this.

My recommendations to anybody interested in pursuing practical research in ethnomathematics would be to design a study that could consciously be used to help participants understand and change their situations.

Epilogue

Once in a while, one meets somebody who has a genuinely heartfelt regard for knowledge and a love of teaching. Every good educational institution has at least a few teachers who would be described thus.

Mr. S was such a person. His classroom was unconventional—a carpentry workshop. His educational background was unusual for a teacher of such stature—after finishing elementary school, he discontinued his formal education and began an informal apprenticeship with his father. His topic—carpentry, or was it mathematics? (I learned a lot of both from him.)

For 44 years he had been designing and constructing furniture and restoring antiques. And he was an insightful mathematician. Mr. S lived his mathematics. It was a part of his actions, a part of his environment. From his home on the mountainside, he looked out over the city and the harbor. He saw the horizon as the straight line that God gave us, where the sea meets the sky. Continuing the discussion about straight lines, he talked about the rays of the sun, the vertical upward path of the bubbles that emerge from the mouths of the goldfish in his fishtank.... His mathematics is unfortunately not yet found in children's textbooks.

He liked to surround his knowledge with some mystery. That which had great value was not shared lightly. Mutual trust and respect were part of the unspoken bond between teacher and student. Secrecy shrouded some of his teaching. He told me of the legendary worms that hang upside down in caves in Saudi Arabia, whose excreta is collected and turned into shellac for French polishing! He made mysterious potions for coloring wood and for cleaning antiques. His remedies were trusted despite the fact that their contents were unknown.

Along with his love of mystery, Mr. S could demonstrate his insight into a complex idea with a hand gesture, or he could build a physical model to explain a concept. His teaching was vivid, active, inviting, and engaging. My time as his student provided me with unique opportunities to gain profound insights into both the experience of mathematics and the art of teaching.

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APPENDICES

Appendix A: Script for "Semi-structured" Conversations

A. General, open-ended questions to start the person talking about his life.

B. More specific questions (not all of these were asked each person):

1. When did you leave school?
2. Did you learn mathematics at school?
3. Do you think that mathematics has much to do with what goes on in the workshop?
4. How did you begin learning carpentry?
5. Can you remember what it was like to be an apprentice?
6. Can you remember anything you made that turned out particularly well and that you are especially proud of? Why? How was it different from other things you have made?
7. Can you think of any carpenters you have met who do excellent work?
8. Can you think of any carpenters you have met who do poor work?
9. What makes a "good" carpenter? In what ways does he work or think differently from someone who is not good?
10. When you have a problem to solve, do you ask the others for help?
11. What does it take to be a good apprentice?
12. If you could choose an apprentice to teach, what would you look for?
13. What was it about your father's (or the teacher's) teaching that made it special? What does it take to be a good master teacher?
14. When you look at a piece of work, what particular things do you look for when forming an opinion about its quality?

Appendix B: Script for Structured Interviews

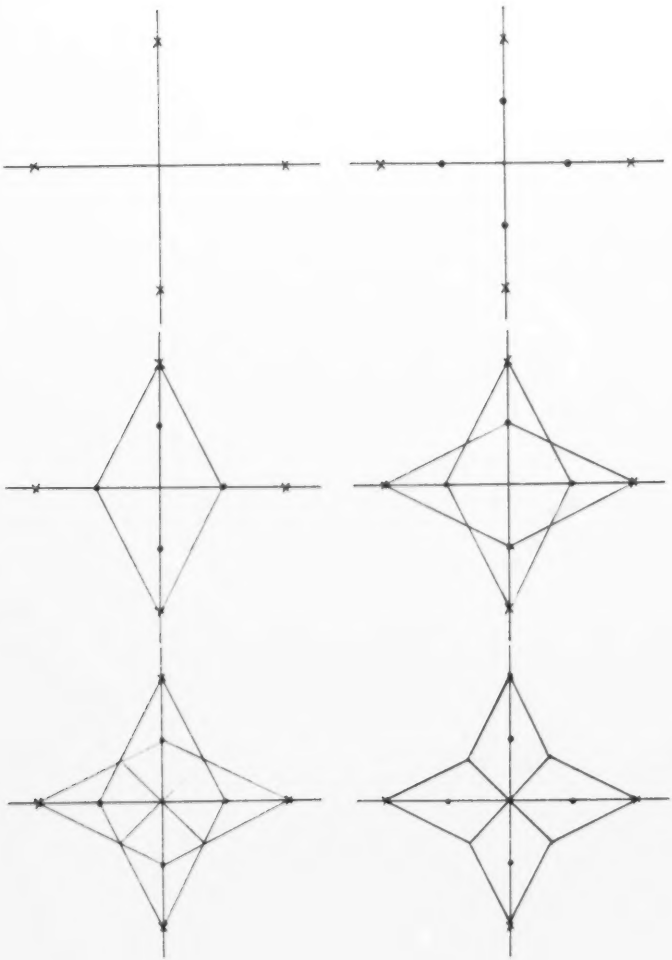
1. Give interviewee a piece of paper cut the same size and shape as the small box ("wakis") that they had all seen me making. Ask how to place the dovetail joints along the side. Look for use of proportion, symmetry, measuring, comparing, angles, arguments for practicality and/or aesthetics. Ask them to comment on my work.

2. Begin a discussion about straight lines. What is a straight line? How do you check that a line is straight? Look for use of symmetries, innovative explanations.

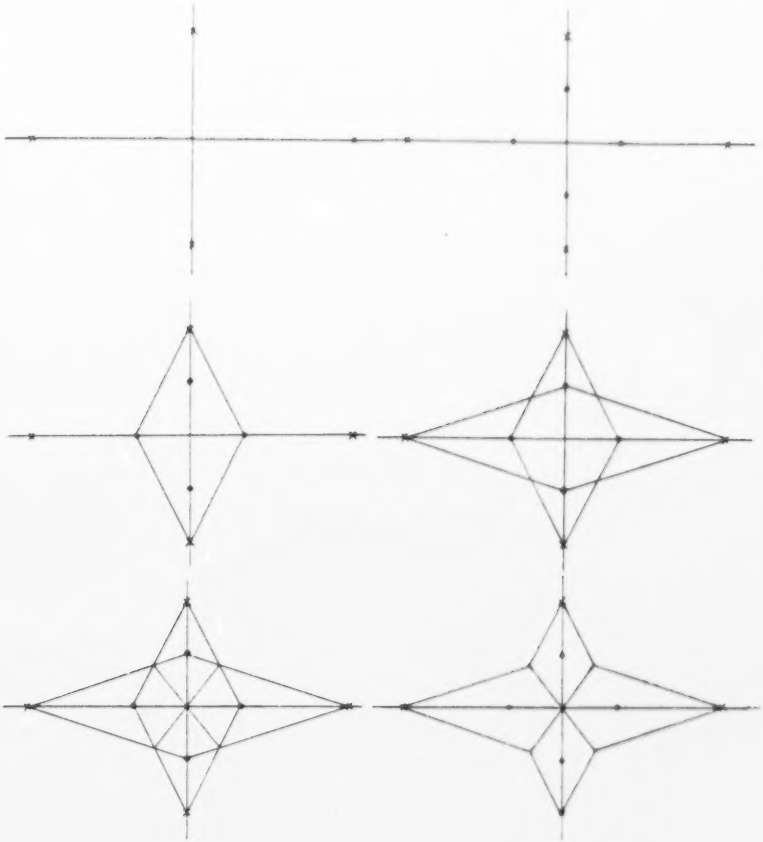
3. What is the meaning of "square"? When is something square? How do you check this?

4. Peter pays R3000 per cubic metre for mahogany. How much does a plank with dimensions 2 meters by 0.6 meters by 0.075 meters cost?

Appendix C: How to Draw Jack's "Square" Star



Appendix D: Jack's Modification of the Steps for a Square Star



Appendix E: Jack's Eight-pointed Star

Note: This is my interpretation and diagram of Jack's verbal explanation. I have presented his explanation as a series of steps.

1. Sketch a four-pointed star as before.

2. Find the center of *AB*, mark this midpoint *D*.

3. Find the center of *BC*, Mark this midpoint *E*.

4. Extend *XA* and *YC*. Join *XY*.

5. With straightedge parallel to *XY*, move straightedge outwards until you have reached the place where you want the extended star point to reach.

6. Draw a line there parallel to *XY* and which cuts *XA* and *YC*.

7. Find the midpoint of this line. Call this midpoint *F*.

8. Join *DF* and *EF*. This makes the first extra star point. Repeat this procedure between each pair of points of the four-pointed star.

PROBLEM: It is easy to "go out of square" with this method.

Appendix F: Mr. S's Eight-pointed Star

Note: This is my interpretation and diagram of Mr. S's verbal description and hasty sketch on a piece of wood.

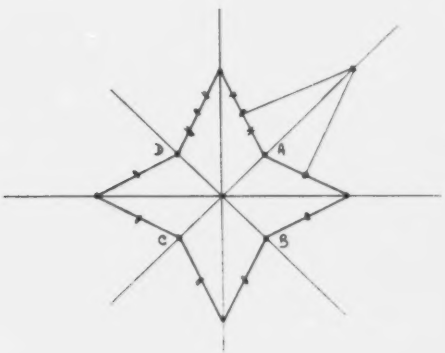
1. Draw a four-pointed star as before.

2. Draw another set of orthogonal axes through the origin, intersecting the four-pointed star at four "inward-pointing" points.

3. Find the midpoints of each of the eight sides of the star.

4. Decide on the length of the new four arms to be added.

5. Mark the positions of the endpoints of the four new arms (of the eight-pointed star) along the second set of orthogonal axes.

6. With straight lines, join each of the four newly marked points to the midpoints of the sides of the four-pointed star that are closest to each new point.

NOTE: In contrast to Jack's method, Mr. S's method ensures that the eight-pointed star remains "square."